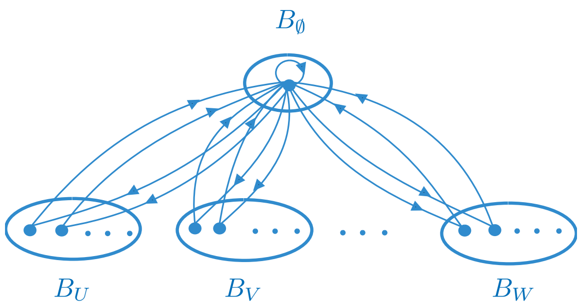
AAA102

102nd Workshop on General Algebra

University of Szeged, Hungary

24–26 JUNE, 2022

$$\mathbb{G}^{\mathbb{H}^* \times \mathbb{K}} \cong \mathbb{H}^{\mathbb{G}^* \times \mathbb{K}}$$



$$\emptyset \neq U, V, \ldots, W \subseteq G^* \times H^*$$
$$|B_U| = |B_V| = \cdots = |B_W| = 2^\kappa$$

BOOK OF ABSTRACTS

PROGRAMME

JUNE 24, FRIDAY

9:00–10:00 **Registration**

	Bolyai Lecture Hall
10:00–10:50	**Zarathustra Brady** Rounding rules and vague solutions to bounded width CSPs

11:00–11:30 **Coffee break**

	Bolyai Lecture Hall
11:30–11:50	**Reinhard Pöschel** Generalized quasiorders and a challenging problem
12:00–12:20	**Erhard Aichinger** Finite representation of higher commutators

12:30–14:00 **Lunch break**

	Bolyai Lecture Hall
14:00–14:50	**Friedrich Martin Schneider** Dynamics of von Neumann's continuous geometries

15:00–15:30 **Coffee break**

	Bolyai Lecture Hall	Szőkefalvi Lecture Hall
15:30–15:50	**Valdis Laan** Injective hulls and completions of ordered algebras	**Endre Tóth** Polymorphism-homogeneous groupoids on the three-element set
16:00–16:20	**Gergő Gyenizse** Lattice medians	**Ganna Kudryavtseva** Proper Ehresmann semigroups

	Bolyai Lecture Hall	Szőkefalvi Lecture Hall
16:30–16:50	**Peter Mayr** Supernilpotent reducts of nilpotent algebras	**Miroslav Korbelář** Congruence-simple semirings of some classes
17:00–17:20	**Patrick Wynne** Clonoids between modules	**Filippo Spaggiari** Mutually normalizing regular subgroups of the holomorph of C_{p^n}
17:30–17:50	**Michael Kompatscher** Finitely based 2-nilpotent Maltsev algebras	**Alvin Lepik** Perfect semigroups

19:00 **CONFERENCE DINNER**

JUNE 25, SATURDAY

	Bolyai Lecture Hall
9:00–9:50	**Luís Oliveira** Regular semigroups weakly generated by idempotents

10:00–10:30 **Coffee break**

	Bolyai Lecture Hall	Szőkefalvi Lecture Hall
10:30–10:50	**Florian Starke** The smallest hard Trees	**Jelena Radović** Commutators in Rees matrix semigroups
11:00–11:20	**Bertalan Bodor** CSPs over structures with slow unlabelled growth	**Bojana Pantić** Model theoretic properties of similarity spaces
11:30–11:50	**Mike Behrisch** On equationally additive clones	**Kristo Väljako** Tensor Product Rings and Morita Equivalence
12:00–12:20	**Bernardo Rossi** Equationally additive constantive Mal'cev clones	**Naqeeb ur Rehman** Cellular Automata on Racks

12:30–14:00 **Lunch break**

	Bolyai Lecture Hall
14:00–14:50	**Miroslav Ploščica** Congruence lattices of Abelian lattice-ordered groups

15:00–15:30 **Coffee break**

	Bolyai Lecture Hall	Szőkefalvi Lecture Hall
15:30–15:50	**Tomas Nagy** Between heaven and hell: The Bodirsky-Pinsker conjecture for hypergraphs	**Branimir Šešelja** Lattice isomorphism of groups
16:00–16:20	**Žaneta Semanišinová** Constraint Satisfaction Problems of First-Order Expansions of Algebraic Products	**Eszter K. Horváth** The number of subuniverses, congruences, weak congruences of semilattices defined by trees
16:30–16:50	**Dmitriy Zhuk** Clones on 3 elements: A New Hope (part I)	**Delbrin Ahmed** $(1+1+2)$-generated lattices of quasiorders
17:00–17:20	**Albert Vucaj** Clones on 3 elements: A New Hope (part II)	**Sándor Radeleczki** Atoms and dual atoms of the lattice of residuated mappings
17:30–17:50	**Erkko Lehtonen** On clonoids of Boolean functions	**Utsithon Chaichompoo** Some Algebraic Properties of Left Groups Which Are Transformation Semigroups
18:00–18:20		**Kritsada Sangkhanan** The regular part of transformation semigroups that preserve double direction equivalence relation

JUNE 26, SUNDAY

	Bolyai Lecture Hall
9:00–9:50	**Ágnes Szendrei** A Characterization of the Commutator in Varieties with a Difference Term

10:00–10:30 **Coffee break**

	Bolyai Lecture Hall
10:30–10:50	**Petar Marković** On the Constraint Satisfaction over SMB algebras
11:00–11:20	**Ádám Kunos** On the substructure ordering of finite directed graphs: definability and automorphisms
11:30–11:50	**David Stanovský** Supernilpotent loops
12:00–12:20	**Péter Pál Pálfy** Commutative conservative clones

INVITED TALKS

Rounding rules and vague solutions to bounded width CSPs

Zarathustra Brady
Massachusetts Institute of Technology, USA

We say that an instance of a constraint satisfaction problem is $1-\epsilon$ solvable if there is an assignment to its variables which satisfies a $1-\epsilon$ fraction of its constraints. Guruswami and Zhou considered the problem of "robustly" solving CSPs: given an instance which is promised to be $1-\epsilon$ solvable, we must find a $1-f(\epsilon)$ solution, for a function $f(\epsilon)$ which approaches 0 as ϵ approaches 0. Barto and Kozik showed that CSP(**A**) is robustly solvable if and only if **A** has bounded relational width (if **A** is a "core", this occurs if and only if the polymorphism clone of **A** generates a congruence meet-semidistributive variety), and they gave an algorithm based on semidefinite programming for which the function f satisfies $f(\epsilon) = O(\log\log(1/\epsilon)/\log(1/\epsilon))$. Traditionally, such algorithms are usually described in terms of "rounding rules" which convert probability distributions over local solutions to a CSP instance into actual solutions, but it is difficult to phrase Barto and Kozik's algorithm in this way.

A rounding rule can be thought of as a family of polymorphisms which satisfy a sufficiently strong system of height 1 identities. We prove that any finite CSP template **A** with bounded width has such a family of polymorphisms, with n-ary polymorphisms corresponding (roughly) to total orderings of the power set of $\{1, ..., n\}$ - we call such an ordering a "vague element" of the set $\{1, ..., n\}$. This collection of polymorphisms can then be used to describe a rounding rule for (approximate) solutions to semidefinite relaxations of CSPs, giving a simpler robust algorithm for CSPs of bounded width. In addition, the corresponding function f satisfies $f(\epsilon) = O(1/\log(1/\epsilon))$, which is known to be best possible (if the Unique Games Conjecture is true) by a result of Guruswami and Zhou.

Regular semigroups weakly generated by idempotents

Luís Oliveira
University of Porto, Portugal

A regular semigroup S is a semigroup where each element x has at least one inverse in von Neumann's sense, that is, an element x' such that $xx'x = x$

and $x'xx' = x'$. Very often, subsemigroups of S are not regular. Thus, the usual concept of variety of algebras is not adequate for these semigroups. For regular semigroups, we consider instead e-varieties, a class of regular semigroups closed for homomorphic images, *regular* subsemigroups and direct products. However, attempts to define a useful notion of free objects for e-varieties have failed to work for all e-varieties. In this talk we will explore the reasons for this to happen.

We say that S is weakly generated by a set A if S has no proper regular subsemigroup containing A. We will explain how a positive answer to the following question may contribute with a different approach to the problem of finding a suitable notion for free objects for all e-varieties: Is there a regular semigroup $F(X)$ weakly generated by X such that all other regular semigroups weakly generated by X are homomorphic images of $F(X)$? The remainder of this talk will be focused on the previous question restricted to the case of X being a set of idempotents. We will see there is a positive answer for this particular case, say $FI(X)$. The semigroup $FI(X)$ will be introduced by a presentation, where both the sets of generators and relations are infinite. Nevertheless, we will see that the word problem for this presentation is decidable as we will present a canonical form for the corresponding congruence classes. If time allows, we will explore the structure of $FI(X)$ and mention some open questions on this topic.

Congruence lattices of Abelian lattice-ordered groups

Miroslav Ploščica
Šafárik University, Slovakia

Compact congruences of Abelian lattice-ordered groups (ℓ-groups) correspond to compact (equivalently, principal) ℓ-ideals. The compact ideals of an ℓ-group G form a distributive lattice $\mathrm{Id}_c\, G$. We discuss the problem of describing the lattices isomorphic to $\mathrm{Id}_c\, G$ for some Abelian ℓ-group G. Equivalently, we are interested in the spectral spaces of Abelian ℓ-groups (i.e. Stone spaces of lattices $\mathrm{Id}_c\, G$).

The solution in the finite case is known for a long time. A finite distributive lattice L is representable as $\mathrm{Id}_c\, G$ if and only if L is *completely normal*, which means that for every $x, y \in L$ there are $u, v \in L$ with $x \vee y = u \vee y = x \vee v$ and $u \wedge v = 0$. (Equivalently, the prime ideals of L form a root system.)

In the infinite case, the complete normality is necessary but not sufficient. A counterexample of the size $\aleph_1$ has been found by Delzell and Madden. Later analysis of this example has lead to the formulation of a new property of lattices $\mathrm{Id}_c\, G$, called *having countably based differences*. This means that for every $x, y \in L$, the ordered set $\{t \in L \mid x \leq y \vee t\}$ has a countable coinitial subset.

The gap $|L| = \aleph_0$ has been filled by Wehrung, who proved that every countable completely normal distributive lattice is representable as $\mathrm{Id}_c\, G$ for some Abelian ℓ-group G.

Another Wehrung's result says that complete normality together with countably based differences are still not sufficient to characterize lattices $\mathrm{Id}_c\, G$. He proved that these lattices satisfy the so-called *Cevian property*, which is a strengthening of complete normality, and constructed a distributive lattice (of the size $\aleph_2$), which is completely normal, has countably based differences, but fails to be Cevian.

We present here another construction (of size continuum plus), of a Cevian distributive lattice with countably based differences, which is not representable as $\mathrm{Id}_c\, G$.

Our another result says that every completely normal distributive lattice of size up to $\aleph_1$ is Cevian. This leads to the problem whether every completely normal distributive lattice of cardinality $\aleph_1$ with countably based differences is isomorphic to some $\mathrm{Id}_c\, G$. We present a partial solution to this problem, by showing that every such lattice is a homomorphic image of some $\mathrm{Id}_c\, G$.

Dynamics of von Neumann's continuous geometries

Friedrich Martin Schneider
Technische Universität Bergakademie Freiberg, Germany

In a series of lectures at Princeton between 1935 and 1937, John von Neumann developed a continuous version of projective geometry: the central objects of this study, continuous geometries, are complete, complemented, modular lattices, whose operations furthermore satisfy a certain continuity property. Beyond classical finite-dimensional projective geometries, the class of continuous geometries contains, for instance, every orthocomplemented complete modular lattice, as well as the projection lattice of any finite von Neumann algebra.

In the course of his analysis, von Neumann established the following remarkable coordinatization theorem: every complemented modular lattice (with order at least four) is isomorphic to the lattice of principal left ideals of some (up to isomorphism unique) regular ring. Furthermore, he proved that every irreducible continuous geometry possesses a unique dimension function (with values in the closed real unit interval), which then induces a compatible complete metric on the corresponding coordinatizing ring and thus furnishes the latter with a natural topology. The topological groups of units of such "continuous rings" exhibit very peculiar dynamical behavior.

In the talk, I will give a brief overview of von Neumann's continuous geometry and report on some recent progress in understanding the structure and dynamics of topological subgroups of unit groups of continuous rings.

A Characterization of the Commutator in Varieties with a Difference Term

Ágnes Szendrei
University of Colorado Boulder, USA

In [2], Emil Kiss introduced 4-ary difference terms for congruence modular varieties, and used them to characterize $[\alpha, \beta] = 0$. I will discuss the ideas of a proof in [1] showing that this characterization extends to varieties with a difference term.

[1] Kearnes, K. A., Szendrei, Á., Willard, R.: Characterizing the commutator in varieties with a difference term. *Algebra Universalis* **83**, article no. 17 (2022)

[2] E. W. Kiss: Three remarks on the modular commutator. *Algebra Universalis* **29**(4), 455–476 (1992)

CONTRIBUTED TALKS

$(1+1+2)$-generated lattices of quasiorders

Delbrin Ahmed
University of Szeged, Hungary

A lattice is $(1+1+2)$-generated if it has a four-element generating set such that exactly two of the four generators are comparable. Based on our joint paper [1], we prove that the lattice Quo(n) of all quasiorders (also known as preorders) of an n-element set is $(1+1+2)$-generated for $n = 3$ (trivially), $n = 6$ (when Quo(6) consists of 209 527 elements), $n = 11$, and for every natural number $n \geq 13$. Except for Quo(6), an extension of Zádori's method is used.

Joint work with Gábor Czédli.

[1] Ahmed, D. and Czédli, G.: (1+1+2)-generated lattices of quasiorders; Acta Sci. Math. (Szeged) **87** (2021), 415–427.

[2] Zádori, L.: Generation of finite partition lattices. Lectures in universal algebra (Proc. Colloq. Szeged, 1983), Colloq. Math. Soc. János Bolyai, Vol. **43**, North-Holland, Amsterdam, 1986, pp. 573–586.

Finite representation of higher commutators

Erhard Aichinger
Johannes Kepler University Linz, Austria

The higher commutators of a finite algebra $\mathbf{A}$ (defined by A. Bulatov) can be seen as a function $C : \bigcup_{n\in\mathbb{N}}(\mathrm{Con}(\mathbf{A}))^n \to \mathrm{Con}(\mathbf{A})$. In this talk, we will exhibit certain finite representations of C and discuss how they can be computed.

Joint work with Nebojša Mudrinski (Novi Sad) and supported by the Austrian Science Fund (FWF):P33878.

On equationally additive clones

Mike Behrisch
TU Wien / JKU Linz, Austria

For $n \in \mathbb{N}$, a relation $\rho \subseteq A^n$ is *algebraic* over a clone F on A if there is a set I and $f_i, g_i \in F^{(n)}$ for $i \in I$ such that

$$\rho = \{x \in A^n \mid \forall i \in I\colon f_i(x) = g_i(x)\}.$$

A clone F on A is *equationally additive* if the union of any two of its algebraic relations of the same arity is again algebraic. In the talk we shall describe all equationally additive Boolean clones, and we shall investigate the number of equationally additive clones on larger finite sets.

Joint work with Erhard Aichinger and Bernardo Rossi (JKU Linz) and funded by FWF project P33878.

CSPs over structures with slow unlabelled growth

Bertalan Bodor
Charles University Prague, Czech Republic

For a structure $\mathfrak{A}$ we denote by $f_n(\mathfrak{A})$ the number of orbits of the natural action of $\mathrm{Aut}(\mathfrak{A})$ on the n-element subsets of A. Let us denote by $\mathcal{S}$ the class of all countable structures for which there exists an $\varepsilon > 0$ such that $f_n(\mathfrak{A}) < (2-\varepsilon)^n$ if n is large enough. In my talk I present a complete classification of structures in $\mathcal{S}$ in terms of their automorphism groups. As a consequence of this classification we can show that all structures in $\mathcal{S}$ are first-order interdefinable with a finitely bounded homogeneous structure. This means that all these structures fall into the scope of the infinite domain CSP dichotomy conjecture stated by Bodirsky and Pinsker.

In this talk I will present a proof of this conjecture for all stable structures in the class $\mathcal{S}$ and discuss some of the difficulties one runs into in the unstable case. I will also discuss some other known results and conjectures for some generalizations of the class $\mathcal{S}$.

Some Algebraic Properties of Left Groups Which Are Transformation Semigroups

Utsithon Chaichompoo
Chiang Mai University, Thailand

Let V be a vector space over a field and $L(V)$ a semigroup of all linear transformations from V to V under composition of functions. For a fixed subspace U of V, define a subsemigroup $Q_U(V)$ of $L(V)$ by

$$Q_U(V) = \{\alpha \in L(V) : V = \ker\alpha \oplus U \quad \text{and} \quad \operatorname{im}\alpha = U\}.$$

In this work, we will show that $Q_U(V)$ is a left group and can be written as a union of general linear groups. Then Green's relations and isomorphism conditions are characterized. Moreover, we compute the rank of $Q_U(V)$ when V is a finite-dimensional vector space over a finite field. Finally, we describe left groups which can be embedded in $Q_U(V)$.

Joint work with Kritsada Sangkhanan.

Lattice medians

Gergő Gyenizse
University of Szeged, Hungary

Let $\mathbf{L}$ be a lattice. A mapping $f : L^3 \to L$ is called a *lattice median* if it is a symmetrical monotone majority operation, and an *inner lattice median* if it is furthermore a lattice term function. The medians of $\mathbf{L}$ form a lattice that is in the variety generated by $\mathbf{L}$, and the inner medians form a sublattice of it. This sublattice is also a homomorphic image of SL(3), the sublattice of the 3-generated free lattice consisting of symmetrical terms.

The number of subuniverses, congruences, weak congruences of semilattices defined by trees

Eszter K. Horváth
University of Szeged, Hungary

The number of subuniverses of semilattices defined by arbitrary and special kinds of trees will be given via combinatorial considerations. Using a result of Freese and Nation, a formula will be given for the number of congruences of semilattices defined by arbitrary and special kinds of trees. Some interesting properties of the congruence lattice of a semilattice corresponding to a tree will be given. Using the number of subuniverses and the number of congruences, a formula will be given for the number of weak congruences of semilattices defined by a binary tree. Some special cases will be discussed. The solution of two apparently nontrivial recurrences will be presented.

Joint work with Delbrin Ahmed and Zoltán Németh.

Finitely based 2-nilpotent Maltsev algebras

Michael Kompatscher
Charles University Prague, Czech Republic

An algebra is called *finitely based*, if the variety it generates is axiomatizable by finitely many identities. While it is known that e.g. finite groups are finitely based, there is no classification of finitely based Maltsev algebras (i.e. algebras having a Maltsev term $m(y, x, x) \approx m(x, x, y) \approx y$) in general. In particular, nilpotent, but not-supernilpotent Maltsev algebras are not covered by known results.

In this talk we present some examples of finitely based 2-nilpotent Maltsev algebras. We additionally show that their subpower membership problem is solvable in polynomial time.

Congruence-simple semirings of some classes

Miroslav Korbelář
Czech Technical University in Prague, Czech Republic

Congruence-simple semirings are studied as the simplest case of subdirectly irreducible semirings. Recently, they gained also a lot of attention thanks to their possible applications in cryptography. A complete classification of commutative congruence-simple semirings was done by El Bashir et al, and a great contribution to the classification of finite (generally non-commutative) cases was provided by Kendziorra and Zumbrägel. We study congruence-simple semirings with a multiplicatively absorbing element and provide a basic classification of further classes - those ones that either have no non-trivial nilpotents or that are multiplicatively idempotent. We also characterize all multiplicatively idempotent and congruence-simple semirings that are finite.

Joint work with T. Kepka and G. Landsmann.

Proper Ehresmann semigroups

Ganna Kudryavtseva
University of Ljubljana, Slovenia

We propose a notion of a proper Ehresmann semigroup based on a three-coordinate description of its generating elements governed by certain labelled directed graphs with additional structure. The generating elements are determined by their domain projection, range projection and σ-class, where σ denotes the minimum congruence that identifies all projections. We prove a structure result on proper Ehresmann semigroups and show that every Ehresmann semigroup has a proper cover. Our covering semigroup turns out to be isomorphic to that from the work by Branco, Gomes and Gould and provides a new view of the latter. Proper Ehresmann semigroups all of whose elements admit a three-coordinate description are characterized in terms of partial multiactions of monoids on semilattices. As a consequence we recover the two-coordinate structure result on proper restriction semigroups.

Joint work with Valdis Laan.

On the substructure ordering of finite directed graphs: definability and automorphisms

Ádám Kunos
University of Szeged, Hungary

In 2009–2010 J. Ježek and R. McKenzie published a series of papers in which they examined the first-order definability in the substructure orderings of finite mathematical structures with a given type. They considered finite semilattices, ordered sets, distributive lattices, and lattices. Their results were analogous for all these structure types. Since then, some have picked up this line of research—including the author of the present talk. There has been a certain pattern in the aforementioned results. For example, the automorphism groups of the orderings always turned out to be almost trivial, i. e. isomorphic to the trivial group or the two element one. This pattern breaks for the substructure ordering of finite directed graphs. In this talk, we tell a conjecture, naming a 768-element candidate for the automorphism group, which, for the breaking pattern, will require a new method to prove. Though incomplete, we show a path for a possible proof.

Joint work with Fanni K. Nedényi.

Injective hulls and completions of ordered algebras

Valdis Laan
University of Tartu, Republic of Estonia

We consider partially ordered universal algebras where all operations are monotone. Using so-called linear functions one can construct a closure for each subset of such an algebra A. The set of all closed subsets will have the structure of a sup-algebra. It turns out that this set of closed subsets is an injective hull of A in a certain category with respect to some specific class of monomorphisms. It can be shown that this injective hull has properties that are similar to the properties of the Dedekind-MacNeille completion of a poset (which is an ordered algebra with no operations). Thereby we generalize a well-known fact that the Dedekind-MacNeille completion of a poset is its injective hull.

Joint work with Xia Zhang, Jianjun Feng and Ülo Reimaa.

On clonoids of Boolean functions

Erkko Lehtonen
Universidade Nova de Lisboa, Portugal

Let C_1 and C_2 be clones on sets A and B, respectively. A set K of functions of several arguments from A to B is called a (C_1, C_2)-*clonoid* if $KC_1 \subseteq K$ and $C_2K \subseteq K$. In 2019, A. Sparks classified the clones C on $\{0, 1\}$ according to the cardinality of the lattice of (J_A, C)-clonoids. (Here J_A denotes the clone of projections on A.) Whether this lattice is finite, countably infinite, or uncountable depends only on whether C contains a majority, Mal'cev, or near-unanimity operation.

Sparks's result was refined in our recent work (partly joint with M. Couceiro), where the (C_1, C_2)-clonoids of Boolean functions were completely described in the cases where the clone C_1 is arbitrary and C_2 contains a majority or a minority function. We also report some new developments on the case where C_2 contains a near-unanimity operation.

Perfect semigroups

Alvin Lepik
University of Tartu, Republic of Estonia

Perfect monoids were initially studied by Isbell and Fountain. Their works have inspired other articles focusing on perfection of monoids and pomonoids.

We consider a more general approach, regarding perfection of semigroups and obtain generalisations to the class of (factorisable) semigroups for many known results in the case of monoids. Our results also allow us to conclude that completely zero simple semigroups are perfect and that perfection is a Morita invariant for factorisable semigroups.

On the Constraint Satisfaction over SMB algebras

Petar Marković
University of Novi Sad, Serbia

In this talk I will introduce the variety of semilattices of Mal'cev blocks (SMB algebras). SMB algebras are Taylor algebras which can be found quite often in any finite Taylor algebras as factors of subalgebras of term reducts. Bulatov first proved the tractability of the Constraint Satisfaction Problem (CSP) over SMB algebras and then went on to generalize the algorithm to all Taylor algebras, thus proving the Dichotomy Conjecture of the CSP. I will compare the Dichotomy proofs of Zhuk and Bulatov on SMB algebras and present a way to combine them to possibly find a proof which is simpler than either.

Joint work with M. Maróti, R. McKenzie and A. Prokić.

Supernilpotent reducts of nilpotent algebras

Peter Mayr
University of Colorado Boulder, USA

We show that every finite nilpotent Mal'cev algebra has a supernilpotent Mal'cev reduct, that is, a reduct that is a direct product of algebras of prime power order.

Recall that every finite supernilpotent Mal'cev algebra is finitely based by work of Vaughan–Lee (1983) and Freese and McKenzie (1987). In his paper Vaughan–Lee also points out a particular nilpotent loop of size 12, which is not supernilpotent and hence not covered by their techniques. Using its supernilpotent reduct, we can now show that this loop is still finitely based.

Joint work with Michael Kompatscher and Patrick Wynne.

Between heaven and hell: The Bodirsky-Pinsker conjecture for hypergraphs

Tomas Nagy
Technische Universität Wien, Austria

The recently developed algebraical theory of smooth approximations allows us for the first time to classify the complexity of large classes of infinite-domain CSPs. Using this theory, we can reduce the problem of finding an injective solution to a CSP over random k-uniform hypergraph to a finite-domain CSP.

Surprisingly, it shows up that the problem of finding a general solution to such CSPs cannot be reduced to a finite-domain CSP in a natural way – the reason for this is that the behavior of polymorphisms of templates under consideration can depend on a linear order. This forces us to develop new algorithmic techniques in order to reduce the problem of finding a general solution to such CSPs to the problem of finding an injective solution. This way, we confirm the Bodirsky-Pinsker conjecture for the random k-uniform hypergraphs for any k.

Joint work with Antoine Mottet and Michael Pinsker.

Commutative conservative clones

Péter P. Pálfy
Alfréd Rényi Institute of Mathematics, Hungary

A clone C of operations on a set A is called commutative if every $f \in C$ is a homomorphism $(A; C)^n \to (A; C)$, where n is the arity of f. It is called conservative if every subset of A is a subalgebra. A straightforward application of Zorn's lemma yields that every commutative conservative clone is contained in a maximal one. We have obtained that there are two types of maximal commutative conservative clones.

Type 1: Let $<$ be a linear order on A. For each proper down-set $D \subset A$ let

$$f_D(a_1, a_2) = \begin{cases} a_1, & \text{if } a_1, a_2 \in D; \\ \max(a_1, a_2), & \text{otherwise.} \end{cases}$$

Then $\langle f_D : D \subset A \text{ down-set} \rangle$ is a maximal commutative conservative clone.

Type 2: Let $m_1, m_2 \in A$, and let $<$ be an almost linear order on A, in the sense that m_1 and m_2 are minimal elements with respect to $<$, and this is the only pair of incomparable elements. Let

$$f^{(3)}(a_1,a_2,a_3) = \begin{cases} \text{minority}(a_1,a_2,a_3), & \text{if } a_1,a_2,a_3 \in \{m_1,m_2\}; \\ \max(a_1,a_2,a_3), & \text{otherwise.} \end{cases}$$

Then $\langle f^{(3)}, f_D : \{m_1,m_2\} \subset D \subset A$ down-set$\rangle$ is a maximal commutative conservative clone.

This work is a continuation of an earlier joint paper with Hajime Machida (Self-commuting operations, *J. Multiple-valued Logic and Soft Computing* 36 (2021), 383–390).

Model theoretic properties of similarity spaces

Bojana Pantić
University of Novi Sad, Serbia

Metric spaces are a very well studied class of structures. Recently, we considered a new class of structures which arose in quite a natural way out of the former one, namely the *similarity spaces*. Such a space consists of a set of points which is equipped with a certain prechain relation called *similarity*.

In this talk I will be reporting on our first findings regarding these structures.

Joint work with Maxime Gheysens, Christian & Maja Pech, and Friedrich Martin Schneider.

Generalized quasiorders and a challenging problem

Reinhard Pöschel
Technische Universität Dresden, Germany

Equivalence relations or, more general, quasiorders (i.e., reflexive and transitive binary relations) ρ have the property that an n-ary operation f preserves ρ (i.e., f is a polymorphism of ρ) if and only if each unary polynomial function obtained from f (by substituting constants) preserves ρ (i.e., is an endomorphism of ρ). We introduce a wider class of relations (called generalized quasiorders) of arbitrary arities with the same property. With these generalized quasiorders we can characterize all algebras whose clone of term operations is determined by its unary polynomial operations by the above property (what generalizes affine complete algebras). We conclude with a challenging problem.

Joint work with Danica Jakubíková-Studenovská and Sándor Radeleczki

Atoms and dual atoms of the lattice of residuated mappings

Sándor Radeleczki
University of Miskolc, Hungary

The residuated mappings of a complete lattice can be defined as complete join-preserving maps from L to L. They form a complete lattice $\mathrm{Res}(L)$ with respect to the point-wise defined order relation. We describe the dual atoms of this lattice, and its atoms in some particular cases. For instance, we prove that for any (complete) pseudocomplemented lattice L the atoms of $\mathrm{Res}(L)$ are weakly regular in sense of Blyth and Janowitz. We characterize the weakly regular atoms of $\mathrm{Res}(L)$ for an arbitrary complete lattice.

Joint work with Kalle Kaarli.

Commutators in Rees matrix semigroups

Jelena Radović
University of East Sarajevo, Bosnia and Herzegovina /
University of Novi Sad, Serbia

We study the centralizing condition and commutators on Rees matrix semigroups. We obtain a complete characterization of the binary commutator on Rees matrix semigroups, and use it to study other properties of the commutator.

Joint work with Nebojša Mudrinski.

Cellular Automata on Racks

Naqeeb ur Rehman
Allama Iqbal Open University, Pakistan

Cellular automata are discrete dynamical systems originally studied by John von Neumann and Stanislaw Ulam in 1950s on cell spaces of integer lattices. Motivated by the modern cellular automata theory (in [1]), the basics of cellular automata on cell spaces of racks (self-distribitive algebraic structures) will be discussed in this talk.

[1] T. Ceccherini-Silberstein, M. Coornaert, Cellular Automata and Groups, Springer Monographs in Mathematics, Springer-Verlag, Berlin (2010).

Equationally additive constantive Mal'cev clones

Bernardo Rossi
JKU Linz, Austria

A clone C is called *equationally additive* if the union of two C-algebraic sets is again a C-algebraic set. We characterize the Mal'cev algebras whose clone of polynomial functions is equationally additive in terms of properties of the binary term condition commutator.

Joint work with E. Aichinger and M. Behrisch. Supported by the Austrian Science Fund (FWF): P33878

The regular part of transformation semigroups that preserve double direction equivalence relation

Kritsada Sangkhanan
Chiang Mai University, Thailand

Let $T(X)$ be the full transformation semigroup on a set X under the composition of functions. For any equivalence relation E on X, define a subsemigroup $T_{E^*}(X)$ of $T(X)$ by

$$T_{E^*}(X) = \{\alpha \in T(X) : \text{for all } x, y \in X, (x, y) \in E \Leftrightarrow (x\alpha, y\alpha) \in E\}.$$

In this work, we show that the regular part of $T_{E^*}(X)$, denoted $Reg(T)$, is the largest regular subsemigroup of $T_{E^*}(X)$. Then its Green's relations and ideals are described. Moreover, we find the kernel of $Reg(T)$ which is a right group and can be written as a union of symmetric groups. Finally, we prove that every right group can be embedded in that kernel.

Constraint Satisfaction Problems of First-Order Expansions of Algebraic Products

Žaneta Semanišinová
Technische Universität Dresden, Germany

We study the complexity of infinite-domain constraint satisfaction problems (CSPs). Our basic setting is that a complexity classification for the CSPs of first-order expansions of a structure $\mathfrak{A}$ can be transferred to a classification of the CSPs of first-order expansions of another structure $\mathfrak{B}$. We exploit the algebraic product of structures that corresponds to the product of the respective polymorphism clones and present a complete complexity classification of the CSPs for first-order expansions of finite algebraic powers of $(\mathbb{Q}; <)$. By combining our classification result with classification transfer techniques, we answer several open questions in spatial reasoning from (Balbiani, Condotta, 2002), (Balbiani, Condotta, del Cerro, 1999) and (Balbiani, Condotta, del Cerro, 2002).

Joint work with Manuel Bodirsky, Peter Jonsson, Barnaby Martin, Antoine Mottet.

Lattice isomorphism of groups

Branimir Šešelja
University of Novi Sad, Serbia

We present some new results related to the lattice characterization of groups. We use the weak congruence lattice of a group G (Wcon(G)), which is a lattice extension of the subgroup lattice (Sub(G)): it consists of all normal subgroups of all subgroups of G, represented by the corresponding congruences on subgroups. If G and G_1 are groups and there is a lattice isomorphism from Sub(G) to Sub(G_1), then these groups are said to be lattice isomorphic, or that there is a projectivity from G to G_1. We extend this notion to w-projectivity, so that subgroup lattices are replaced by lattices of weak congruences. This extension is related to the uniqueness of the element which in Wcon(G) represents the diagonal relation of G. We prove that in many classes of groups (Dedekind, simple, symmetric, ...) this lattice element is unique, being a condition under which w-projectivity implies projectivity. As a consequence, we also give necessary and sufficient conditions which should be satisfied by Wcon(G) so that the group G is an SN-(SI)-group (possessing a normal (invariant) series with abelian factors).

Joint research with A. Tepavčević, J. Jovanović and M. Grulović.

Mutually normalizing regular subgroups of the holomorph of C_{p^n}

Filippo Spaggiari
Charles University Prague, Czech Republic

Let $G = C_{p^n}$ be a finite cyclic $p-$group. We want to find and characterize the regular subgroups of its holomorph that are mutually normalizing each other in the permutation group $Sym(G)$. We represent such regular subgroups as vertices of a graph, and we connect a pair of them by an edge when they mutually normalize each other. The approach to constructing this *local normalizing graph* relies on the theory of *gamma functions*, which is directly related to the structure of set-theoretic solutions of the Yang-Baxter Equation. This new language allows us to reformulate and present several results in an elegant form, difficult to obtain with pure Group Theory. The outcome is a characterization of the graph structure by using simple modular arithmetic presented in a compact form.

Supernilpotent loops

David Stanovský
Charles University Prague, Czech Republic

We will present our initial observations on the concept of supernilpotence for loops, and on how it relates to the classical concepts of central nilpotence and nilpotence of the multiplication group.

The smallest hard Trees

Florian Starke
Technische Universität Dresden, Germany

Given a directed graph G, the constraint satisfaction problem (CSP) of G is the computational problem of deciding whether a finite input graph H has a homomorphism to G. In this talk I will consider finite orientations of trees and the complexity of their CSPs. I will present

- all smallest trees whose CSP is NP-complete (assuming P $\neq$ NP),
- the smallest trees whose CSP is NL-hard (assuming L $\neq$ NL),
- the smallest tree whose CSP cannot be solved by arc consistency,
- the smallest tree whose CSP cannot be solved by Datalog, and
- the smallest tree whose CSP has unknown complexity.

These trees have been found using a computer program. In the talk I will present core ideas of this program.

Polymorphism-homogeneous groupoids on the three-element set

Endre Tóth
University of Szeged, Hungary

Our current goal is to determine which three-element groupoids are polymorphism-homogeneous. There are 19683 groupoids on the three-element set, but up to isomorphism their number is only 3330. In the article [2] Joel Berman and Stanley Burris investigate multiple properties of these groupoids with the help of computers. We continue this path by using a computer to help us make it clearer which groupoids are polymorphism-homogeneous. Naturally, some conjectures arise from the result given by our program(s). We focus on these conjectures, and would like to give some general condition for a groupoid to be polymorphism-homogeneous.

Joint work with Tamás Waldhauser.

[1] E. Tóth and T. Waldhauser, Polymorphism-homogeneity and universal algebraic geometry, Discrete Mathematics & Theoretical Computer Science, special issue in honour of Maurice Pouzet **23**:(2), 2022

[2] J. Berman and S. Burris, A computer study of 3-element groupoids, In Logic and Algebra, (The Proceedings of the Magari Conference), 379 – 430, Marcel Dekker, Inc, 1996

Tensor Product Rings and Morita Equivalence

Kristo Väljako
University of Tartu, Republic of Estonia

My talk will be based on the article "Tensor product rings and Rees matrix rings" (published in Comm. in Algebra).

In my talk I will intriduce tensor product rings $Q \otimes_R P$, where Q_R and ${}_R P$ are R-modules for an arbitrary associative ring R. I will define pseudo-surjective mappings and show that if S is an idempotent ring then a pseudo-surjectively defined tensor product ring $Q \otimes_S P$ is Morita equivalent to S. Then I will define locally injective homomorphisms of rings and strict local isomorphisms of rings. It turns out that a locally injective homomorphism of rings arises naturally from any Morita context between rings.

Finally I will give a new necessary and sufficient condition for firm rings to be Morita equivalent. Also I will give a necessary and sufficient condition for s-unital rings to be Morita equivalenve, which is a generalization result that two R and S with identity are Morita equivalent if and only if there exists a progenerator Q_R with $S \cong \mathrm{End}(Q_R)$.

Clones on 3 elements: A New Hope (part II)

Albert Vucaj
Technische Universität Wien, Austria

In 1959 Janov and Mučnik proved that there exists a continuum of clones over a k-element set for $k \geq 3$. The goal to achieve a classification à la Post for clones on a three-element set seemed to falter. Subsequent research in universal algebra therefore focused on understanding particular aspects of clone lattices on finite domains, for example on the description of *maximal clones* or *minimal clones*. One might still hope to classify all operation clones on finite domains up to some equivalence relation so that equivalent clones share many of the properties that are of interest in universal algebra. Recently, Barto, Opršal, and Pinsker introduced a weakening of the notion of clone homomorphism on the class of clones on a finite set, known as *minor-preserving map*. In this talk we focus on the poset that arises by considering clones on a three-element set with the following order: we set $\mathcal{C} \leq \mathcal{D}$ if there exist a minor-preserving map from $\mathcal{C}$ to $\mathcal{D}$. In particular, we show that the aforementioned poset has only three submaximal elements.

Joint work with Manuel Bodirsky and Dmitriy Zhuk.

The Hochschild-Serre Sequence in Varieties with a Difference Term

Alexander Wires
Southwestern University of Finance and Economics, People's Republic of China

In a fixed signature, we develop 1^{st} and 2^{nd} abelian cohomology groups characterizing extensions in arbitrary varieties realizing affine datum. The resulting cohomology theory admits a semidirect product and a characterization of derivations by stabilizing automorphisms; in addition, this cohomology accepts an additional parameter controlling the equational theories of extensions and so yields a mapping from the lattice of subvarieties to the lattice of cohomology subgroups.

In varieties with a difference term, 2^{nd}-cohomology with trivial actions characterize central extensions. In this general setting, we establish the low-dimensional Hochschild-Serre exact sequence and develop results connected to lifting homomorphisms, covers, possible generalizations of Schur Multipliers, and characterizing 2^{nd}-cohomology by datum algebras derived from relative commutator presentations. By specialization, this recovers the classic constructions from group theory and very recent results concerning algebras of Loday or varieties of modules expanded by multilinear maps.

Clonoids between modules

Patrick Wynne
University of Colorado at Boulder, USA

A clonoid from an algebra $\mathbb{A}$ to an algebra $\mathbb{B}$ is a set of functions from powers of A into B that is closed with respect to precomposition with term functions of $\mathbb{A}$ and closed with respect to postcomposition with term functions of $\mathbb{B}$. We investigate clonoids between finite modules. These structures arise in the description of nilpotent Mal'cev algebras. We show that if the two modules have orders that are not coprime then there are infinitely many clonoids between them. On the other hand, if the modules are of coprime order then the number of clonoids from $\mathbb{A}$ to $\mathbb{B}$ depends on the structure of $\mathbb{A}$. We classify modules over commutative rings for which the clonoids are generated by their unary functions.

Joint work with Peter Mayr.

On free left n-dinilpotent strong doppelsemigroups

Anatolii V. Zhuchok
Luhansk Taras Shevchenko National University, Ukraine

A *doppelsemigroup* (Zhuchok, A. V. (2017). Free products of doppelsemigroups. Algebra Univ., 77(3), 361–374) is a nonempty set D with two binary associative operations $\dashv$ and $\vdash$ satisfying the axioms $(x \dashv y) \vdash z = x \dashv (y \vdash z)$ and $(x \vdash y) \dashv z = x \vdash (y \dashv z)$. A doppelsemigroup $(D, \dashv, \vdash)$ is called *strong* (Zhuchok, A. V. (2018). Structure of free strong doppelsemigroups. Commun. Algebra, 46:8, 3262–3279) if it satisfies the axiom $x \dashv (y \vdash z) = x \vdash (y \dashv z)$. A doppelsemigroup $(D, \dashv, \vdash)$ is called *left dinilpotent* (Zhuchok, A. V. (2017). Free left n-dinilpotent doppelsemigroups, Commun. Algebra, 45:11, 4960–4970) if for some $n \in \mathbb{N}$ and any $x_1, ..., x_n$, $x \in D$ the following identities hold:

$$(x_1 *_1 ... *_{n-1} x_n) \dashv x = x_1 *_1 ... *_{n-1} x_n = (x_1 *_1 ... *_{n-1} x_n) \vdash x,$$

where $*_1, ..., *_{n-1} \in \{\dashv, \vdash\}$. The least such n is called the *left dinilpotency index* of $(D, \dashv, \vdash)$. For $k \in \mathbb{N}$ a left dinilpotent doppelsemigroup of left dinilpotency index $\leq k$ is said to be *left k-dinilpotent*. If ρ is a congruence on a doppelsemigroup $(D, \dashv, \vdash)$ such that $(D, \dashv, \vdash)/\rho$ is a left n-dinilpotent doppelsemigroup, we say that ρ is a *left n-dinilpotent congruence*.

We construct the free object in the variety of left n-dinilpotent strong doppelsemigroups and characterize the least left n-dinilpotent congruence on the free strong doppelsemigroup.

Clones on 3 elements: A New Hope (part I)

Dmitriy Zhuk
Charles University Prague, Czech Republic

In 2017 Moiseev found 2 079 040 clones on 3 elements definable by binary relations on a computer. It is clear that only computers can deal with so many clones but another result from 2019 showed that even a computer has its limitations. Moore proved that it cannot check whether a clone given by generating operations is finitely related (definable by a relation). These results made us believe that even a computer description of all the clones on 3 elements would never appear.

However, not all clones are essentially different, and one might try to characterise clones modulo clone homomorphism or minor preserving map (only h1-identities are preserved). In 2022 we showed that there are only countably many clones of self-dual operations on 3-elements modulo minor preserving map. Then using a computer we collapsed 2 079 040 into a small number, and now we hope that a complete characterization of all clones on 3 elements modulo minor preserving map is possible.

Joint work with Libor Barto, Jan Adam Zahálka, and Albert Vucaj.

LIST OF PARTICIPANTS

Delbrin Ahmed,
University of Szeged, Hungary

Erhard Aichinger,
Johannes Kepler University Linz, Austria

Mike Behrisch,
Technische Universität Wien / Johannes Kepler University Linz, Austria

Bertalan Bodor,
Charles University Prague, Czech Republic

Zarathustra Brady,
Massachusetts Institute of Technology, MA, USA

Utsithon Chaichompoo,
Chiang Mai University, Thailand

Jelena Čolić Oravec,
University of Novi Sad, Serbia

Miklós Dormán,
University of Szeged, Hungary

István Földes,
University of Miskolc, Hungary

Gergő Gyenizse,
University of Szeged, Hungary

Eszter K. Horváth,
University of Szeged, Hungary

Kamilla Kátai-Urbán,
University of Szeged, Hungary

Michael Kompatscher,
Charles University Prague, Czech Republic

Miroslav Korbelář,
Czech Technical University in Prague, Czech Republic

Ganna Kudryavtseva,
University of Ljubljana, Slovenia

Júlia Kulin,
University of Szeged, Hungary

Ádám Kunos,
University of Szeged, Hungary

Valdis Laan,
University of Tartu, Republic of Estonia

Erkko Lehtonen,
Universidade Nova de Lisboa, Portugal

Alvin Lepik,
University of Tartu, Republic of Estonia

Rozália Madarász,
University of Novi Sad, Serbia

László Márki,
Alfréd Rényi Institute of Mathematics, Hungary

Petar Marković,
University of Novi Sad, Serbia

Miklós Maróti,
University of Szeged, Hungary

Peter Mayr,
University of Colorado Boulder, CO, USA

Tomas Nagy,
Technische Universität Wien, Austria

Luís Oliveira,
University of Porto, Portugal

Péter Pál Pálfy,
Alfréd Rényi Institute of Mathematics, Hungary

Bojana Pantić,
University of Novi Sad, Serbia

Miroslav Ploščica,
Šafárik University, Slovakia

Reinhard Pöschel,
Technische Universität Dresden, Germany

Aleksandar Prokic,
University of Novi Sad, Serbia

Sándor Radeleczki,
University of Miskolc, Hungary

Jelena Radović,
University of East Sarajevo / University of Novi Sad, Bosnia and Herzegovina / Serbia

Naqeeb ur Rehman,
Allama Iqbal Open University, Islamabad, Pakistan

Bernardo Rossi,
Johannes Kepler University Linz, Austria

Kritsada Sangkhanan,
Chiang Mai University, Thailand

Friedrich Martin Schneider,
Technische Universität Bergakademie Freiberg, Germany

Žaneta Semanišinová,
Technische Universität Dresden, Germany

Branimir Šešelja,
University of Novi Sad, Serbia

Filippo Spaggiari,
Charles University Prague, Czech Republic

David Stanovský,
Charles University Prague, Czech Republic

Florian Starke,
Technische Universität Dresden, Germany

Csaba Szabó,
Eötvös Loránd University, Hungary

Ágnes Szendrei,
University of Colorado Boulder, CO, USA

Mária B. Szendrei,
University of Szeged, Hungary

Endre Tóth,
University of Szeged, Hungary

Vlado Uljarević,
University of Novi Sad, Serbia

Albert Vucaj,
Technische Universität Wien, Austria

Kristo Väljako,
University of Tartu, Republic of Estonia

Tamás Waldhauser,
University of Szeged, Hungary

Alexander Wires,
Southwestern University of Finance and Economics, People's Republic of China

Patrick Wynne,
University of Colorado at Boulder, CO, USA

László Zádori,
University of Szeged, Hungary

Samir Zahirović,
University of Novi Sad, Serbia

Anatolii Zhuchok,
Luhansk Taras Shevchenko National University, Ukraine

Dmitriy Zhuk,
Charles University Prague, Czech Republic

Organizing committee:

Miklós Dormán
Gergő Gyenizse
Eszter K. Horváth
Kamilla Kátai-Urbán
Júlia Kulin
Ádám Kunos
Miklós Maróti
Mária B. Szendrei
Tamás Waldhauser
László Zádori