The Kakeya Problem over Finite Fields

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Abstract

A Besicovitch set in $\mathbb{R}^n$ is a subset of $\mathbb{R}^n$ containing a unit line segment in every direction and the classical Kakeya conjecture says that every such set has Minkowski and Hausdorff dimension equal to n. The conjecture is true in the case $n = 2$ (Davies 1971) and remains still open for $n > 2$, although many partial results are known (N.Katz, T.Tao, T.Wolff).

It was T.Wolff who firstly considered the Kakeya setting in the finite case. He defined a Kakeya set in $AG(n, q)$ as a point set of $AG(n, q)$ containing a line in every direction [3] and made the celebrated finite field Kakeya conjecture: *for every Kakeya set E in $AG(n, q)$, there exists a constant $c_n > 0$, depending only on n, such that $|E| \geq c_n q^n$.* The conjecture has had a significant influence in the subject of finite field Kakeya set theory and remained open for more than ten years. It was completely solved in 2008 by Z.Dvir using the polynomial method with a beautiful argument [2].

The problem of finding the minimum size of a Kakeya set in $AG(n, q)$ is very hard and is completely solved only in the case $n = 2$ [1].

In this talk, we will give a survey about the previous problems and results.

References

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