On a class of oscillation of not self-adjoint operators associated with differential equations of fractional order

Aleroev, Temirkhan S.

Moscow Institute of a Municipal Services and Construction, Russia

E-mail: aleroev@mail.ru

Aleroeva, H. T.

Moscow Technical Institute of Communications and Informatics

E-mail: BinSabanur@gmail.com

This paper is devoted to the distribution of eigenvalues of operators generated by differential expressions of fractional order with Sturm–Liouville regional conditions. In particular it orders a way of estimating the first eigenvalues.

Geometrical aspects in the study of Lorenz system

Ariesanu Pop, Camelia

Politehnica University of Timişoara, Romania

E-mail: cariesanu@yahoo.com

The chaotic systems have been studied in the last twenty years due to their deep applications in many engineering-oriented applied field, such as nonlinear circuits, synchronization, lasers or secure communications. The first canonical chaotic attractor was found by Lorenz in 1963 and was studied from various points of view.

The goal of our paper is to find some specific values of the parameters for which the Lorenz system admits a Hamilton–Poisson realization and to study the system from mechanical geometry point of view: the phase curves of the system as the intersection between the Hamiltonian and the Casimir of the system, the nonlinear stability problems via energy-Casimir method, periodical orbits, numerical integration via Poisson integrator and numerical simulation.

Laplacian dynamics on graphs with time delays

Atay, Fatihcan M.

Max Planck Institute for Mathematics in the Sciences, Germany

E-mail: atay@member.ams.org

We consider dynamics on finite graphs that evolves under a discrete Laplacian operator subject to time delays. We distinguish between signal-propagation and signal-processing delays, and study the stability of the spatially homogeneous (synchronized) state and its relation to the properties of the underlying graph. Applications are indicated in the fields of the stability of traffic flow, synchronization of coupled oscillators, and consensus in distributed computing and social dynamics.

On the boundedness of the solutions of nonlinear Volterra integral equations

Awwad, Essam
University of Pannonia, Hungary
E-mail: esa_mh@almos.vein.hu

In this paper we study the boundedness property of the solutions of non-linear Volterra integral equation

$$x(t) = \int_0^t f(t, s, x(s))\,\mathrm{d}s + h(t),\ \ t \geq 0,$$

where for any fixed $0 \leq s \leq t$, $f(t, s, \cdot) : \mathbb{R}^d \to \mathbb{R}^d$ and $h(t) \in \mathbb{R}^d$. We give some results on the critical case for the solution of above equation to be bounded. The most important result of this paper is a simple new criterion, which unifies and extends several earlier results. Applications and examples are also given to illustrate our main theorem.

It is a joint work with Prof. István Győri, Department of Mathematics, Faculty of Information Technology, University of Pannonia, Hungary.

Operator splitting for dissipative delay equations

Bátkai, András
Eötvös Loránd University, Hungary
E-mail: batka@cs.elte.hu

We investigate operator splitting methods for a special class of nonlinear partial differential equation with delay. Using results from the theory of nonlinear contraction semigroups in Hilbert spaces, we explain the convergence of the splitting procedure. The order of the convergence is also given in some important, linear special cases.

References

[1] Bátkai, A., Csomós, P., Farkas, B., *Operator splitting for dissipative delay equations*, Preprint, 2011, http://arxiv.org/abs/1009.1981

[2] Bátkai, A., Csomós, P., Farkas, B., Nickel, G., *Operator splitting for non-autonomous evolution equations*, J. Funct. Anal. **260** (2011), 2163–2190, doi:10.1016/j.jfa.2010.10.008

Distribution of delays in infectious disease models

Bélair, Jacques
University of Montreal, Canada
belair@crm.umontreal.ca

Time delays naturally occur as residence times or maturation periods in compartmental models of infectious diseases. These residence time distributions typically take the form either of exponential functions, leading to ordinary differential equations, or fixed constants, leading to delay-differential equations with fixed time lags. We are interested in the influence, if any, of the residence time distribution on, among other dynamical properties, the stability of the equilibria. We present a disease transmission model of SEIRS type with general distributed delays in both the latent and temporary immune periods. We analyse the threshold property of the reproductive number R_0 and the stability properties of the disease-free and endemic equilibria, if any, in the model. Particular attention is paid to the existence of Hopf bifurcations and their dependence on both distributions of delays.

On similarity solutions for boundary layer flows

Bognár, Gabriella
University of Miskolc, Hungary
E-mail: matvbg@uni-miskolc.hu

The existence, uniqueness, non-uniqueness and asymptotic behavior of the solutions to the boundary value problem

$$f''' - \alpha(m){f'}^2 + \beta(m)ff'' = 0$$

$$f(0) = 0, \quad f''(0) = -1, \quad f'(\infty) = 0$$

on R^+ are considered, where $\alpha(m)$ and $\beta(m)$ are rational functions of m. Equations of this type are arising in numerous geophysical systems and industrial processes and are obtained when looking for similarity solutions for boundary layer flows.

We shall study the influence of the parameter m on the properties of the solutions.

On the asymptotic stability of a class of perturbed ordinary differential equations with weak asymptotic mean reversion

Cheng, Jian
Dublin City University, Ireland
E-mail: jian.cheng2@mail.dcu.ie

In this paper we consider the global and local stability and instability of solutions of a scalar nonlinear differential equation with non-negative solutions. The differential equation is a perturbed version of a globally stable autonomous equation with unique zero equilibrium where the perturbation is additive and independent of the state. It is assumed that the restoring force is asymptotically negligible as the solution becomes large, and that the perturbation tends to zero as time becomes indefinitely large. It is shown that solutions are always locally stable, and that solutions either tend to zero or to infinity as time tends to infinity. In the case when the perturbation is integrable, the zero solution is globally asymptotically stable. If the perturbation is non-integrable, and tends to zero faster than a critical rate which depends on the strength of the restoring force, then solutions are globally stable. However, if the perturbation tends to zero more slowly than this critical rate, and the initial condition is sufficiently large, the solution tends to infinity. Moreover, for every initial condition, there exists a perturbation which tends to zero more slowly than the critical rate, for which the solution once again escapes to infinity.

A dynamical systems approach towards isolated vorticity regions for tsunami background states

Constantin, Adrian
University of Vienna, Austria
E-mail: adrian.constantin@univie.ac.at

We prove the existence of two-dimensional tsunami background states with isolated regions of vorticity beneath a flat free surface and surrounded by still water.

A linear oscillator with hysteresis

Csikja, Rudolf
Budapest University of Technology and Economics, Hungary
E-mail: csikja@gmail.com

In technology and science hysteresis is a well known phenomenon. Hysteretic behavior is abundant in nature, and also can be found in almost every households. A simple example is a thermostat. When the temperature reaches a particular (high) point the thermostat switches off and the temperature starts falling down. When another particular (low) point is reached the thermostat switches back on and the temperature starts raising again. Briefly; the thermostat has memory.

Recently the predicted memristor has been realized [1], a fact which is expected to have a big impact on information technology. The memristor intrinsically has memory, hence the name memristor; a resistor with memory.

We investigate a system with two parameters. This system consists of two linear affine oscillators which are hysteretically switched back and and forth.

We construct a two-valued one-dimensional Poincaré-map, which is transformed to a one-valued map by arc-length parametrization. A full bifurcation analysis is given, and also we present some statistical properties of the system.

References

[1] J. J. Yang; M. D. Pickett; X. Li; D. A. A. Ohlberg; D. R. Stewart & R. S. Williams: *Memristive switching mechanism for metal/oxide/metal nanodevices*, Nature Nanotechnology 3, 429–433 (2008).

[2] P. Góra: *Invariant densities for piecewise linear maps of the unit interval,* Ergod. Th. & Dynam. Sys. (2008), XX, 1–10.

[3] R. Csikja, B. M. Garay, J. Tóth: *Chaos via Two-Valued Interval Maps in a Piecewise Affine Model Example for Hysteresis.* 19th International Symposium on Mathematical Theory of Networks and Systems, Budapest, Hungary, 2010.

On the stabilization of the upper equilibrium of the pendulum

Csizmadia, László

University of Szeged, Hungary

E-mail: cslaci@math.u-szeged.hu

Let a be a step-function defined by

$$a(t) = \begin{cases} A_1, & \text{if } k(T_1+T_2) \leq t < (k+1)T_1 + kT_2; \\ -A_2, & \text{if } (k+1)T_1 + kT_2 \leq t < (k+1)(T_1+T_2), \ k \in \mathbb{Z}, \end{cases}$$

where $T_1 > 0$, $T_2 > 0$, $A_1 > 0$, $A_2 > g$ with $A_1T_1 = A_2T_2$, and g denotes the constant of the gravity. Consider the second order differential equation

$$\ddot{x} - \frac{g + a(t)}{l} x = 0,$$

which describes the small vibrations of the l-length pendulum around the upper equilibrium $x = 0$, provided that the suspension point of the pendulum is vibrated vertically by the $T_1 + T_2$-periodic acceleration $a(t)$.

We give a sufficient condition guaranteeing stability for the upper equilibrium. We apply this condition to the classical case $T_1 = T_2$, $A_1 = A_2$, and draw a *global* stability map on the $\varepsilon - \mu$ plane, where

$$\varepsilon^2 := \frac{1}{8}\frac{AT^2}{l}, \quad \mu^2 := \frac{g}{A}.$$

The map is global in the sense that ε and μ are not supposed to be small.

Eventual stability properties of a non-autonomous Lotka–Volterra equation

Dénes, Attila
University of Szeged, Hungary
E-mail: denesa@math.u-szeged.hu

This is a joint work with Professors László Hatvani and László L. Stachó. We investigate different variants of a non-autonomous population dynamical model which describes the change in time of the amount of two fish species – a herbivore and a carnivore – living in Lake Tanganyika and the amount of the plants eaten by the herbivores. The model consists of two parts: reproduction taking place at the end of each year is described by a discrete dynamical system, while the development of the population during a year is described by a non-autonomous system of differential equations that are discussed in the talk. For the first variant of the model we show that the equilibrium of the limit equation of our system (which does not have an equilibrium itself) is a globally eventually uniform-asymptotically stable point of the non-autonomous system. In the proof we use linearization, the method of limit equations and Lyapunov's direct method. For the second variant of the system we show that the limit of each solution of our system is a closed curve which is a solution of the limit equation, which is a Lotka–Volterra equation.

On the critical case in oscillation for differential equations with a single delay and with several delays

Diblík, Josef
Brno University of Technology, Czech Republic
E-mail: diblik@feec.vutbr.cz

The existence of positive solutions of differential equations with delay is often encountered when analysing mathematical models describing various processes. This is a motivation for an intensive study of the conditions for the existence of positive solutions of continuous or discrete equations. Such analysis is related to investigating the case of all solutions being oscillating. We consider a delayed differential linear equation of the form

$$\dot{x}(t) = -a(t)x(t-\tau) \tag{1}$$

where $\tau > 0$ is a constant delay and $a : [t_0, \infty) \to (0, \infty)$. This equation, for its simple form, is often used for testing new results and is very frequently investigated. In the talk, we will deal with sharp conditions for all the solutions of (1) being oscillating and for existence of a positive solution. Results will be modified for equations with several delays. A discrete analogy of (1) will be discussed too.

Maximum principles and stability of delay differential equations

Domoshnitsky, Alexander
Ariel University Center, Israel
E-mail: adom@ariel.ac.il

In this talk we discuss maximum principles for systems of functional differential equations. A connection of maximum principles with nonoscillation and positivity of the Cauchy functions is demonstrated. The method to compare only one component of the solution vector of linear functional differential systems, which does not require heavy sign restrictions on their coefficients, is proposed. Necessary and sufficient conditions of the positivity of elements in a corresponding row of the Cauchy and Green's matrices are obtained in the form of theorems about differential inequalities. The main idea of our approach is to construct a first order functional differential equation for one of the components of the solution vector and then to use assertions about positivity of its Green's functions. This demonstrates the importance to study scalar equations written in a general operator form, where only properties of the operators and not their forms are assumed. It should be also noted that the sufficient conditions, obtained in this talk, cannot be improved in a corresponding sense.

Half-linear differential equations: conditional oscillation and its applications

Došlý, Ondřej
Masaryk University, Czech Republic
E-mail: dosly@math.muni.cz

We consider the half-linear second order differential equation

$$\left(r\left(t\right)\Phi\left(x'\right)\right)' + \lambda c\left(t\right)\Phi\left(x\right) = 0, \quad \Phi\left(x\right) = |x|^{p-2}\, x,\ p > 1, \tag{1}$$

with (positive) continuous functions r, c and the real-valued parameter λ. It is known that the linear oscillation theory (the case $p = 2$ in (1)), in particular, Sturmian separation and comparison theorems, extend verbatim to (1).

Equation (1) is said to be *conditionally oscillatory* if there exists λ_0 such that (1) is oscillatory for $\lambda > \lambda_0$ and nonoscillatory for $\lambda < \lambda_0$. Hence, conditionally oscillatory equations form a natural borderline between oscillatory and nonoscillatory equations. We will report some results of the oscillation theory of (1) where the important role is played just by conditionally oscillatory equations.

Uniform finite difference schemes for singularly perturbed differential difference equations

Erdoğan, Fevzi
Yüzüncü Yıl University, Turkey
E-mail: ferdogan@yyu.edu.tr

The purpose of this study is to present a uniform finite difference method for numerical solution of singularly perturbed differential-difference equations. A fitted difference scheme is constructed in an equidistant mesh, which gives first order uniform convergence in the discrete maximum norm. The method is shown to be uniformly convergent with respect to the perturbation parameter. Some numerical experiment illustrate in practice the result of convergence proved theoretically.

Comparison of different stability and convergence notions in numerical analysis

Faragó, István
Eötvös Loránd University, Hungary
E-mail: faragois@cs.elte.hu

Mincsovics, Miklós
Eötvös Loránd University, Hungary
E-mail: mincso@cs.elte.hu

The classical problem of numerical analysis is the replacement of the original complicated problem by a sequence of problems of simpler structure. Hence the basic question is whether the sequence obtained by solving the simpler problems approaches the solution of the original problem in a suitable way (convergence). Typically, the direct investigation of this question is impossible, therefore we lead the task to the investigation of some other properties that are easier to verify (consistency, stability). The connection between these properties for boundary value problems and initial-boundary value problems is investigated in different ways in the classical numerical theory. In our talk we present a unified investigation of this question for both kinds of problems which include the already existing approaches. On examples of different ordinary and partial differential equations we analyse the results.

Asymptotic behaviour for a Nicholson-type system with patch structure

Faria, Teresa
University of Lisbon, Portugal
E-mail: tfaria@ptmat.fc.ul.pt

For a Nicholson's blowflies model with patch structure and multiple discrete delays, we give conditions for the absolute global asymptotic stability of both the trivial equilibrium and a positive equilibrium (when it exists). The existence of positive heteroclinic solutions connecting the two equilibria is also addressed. We further consider a diffusive Nicholson-type model with patch structure, and establish a criterion for the existence of positive travelling wave solutions, for large wave speeds. Some applications illustrate our results.

Steady states in a structured epidemic model with Wentzell boundary condition

Farkas, József
University of Stirling, UK
E-mail: jzf@maths.stir.ac.uk

We consider a nonlinear structured population model with diffusion. Individuals are structured with respect to a continuous variable which represents for example a pathogen load. The class of uninfected individuals constitutes a special compartment which carries mass, hence the model is equipped with Wentzell (or dynamic) boundary conditions. Our model is intended to describe the spread of infection of a vertically transmitted disease, for example *Wolbachia* in mosquito populations, hence the only nonlinearity arises in the recruitment term. Well posedness of the model and the Principle of Linearised Stability follow from standard semilinear theory. In our main result we establish existence of non-trivial steady states to the model. Our method utilizes an operator theoretic framework with a fixed point approach.

Half-linear differential equations: Two-parametric conditional oscillation

Fišnarová, Simona
Mendel University in Brno, Czech Republic
E-mail: fisnarov@mendelu.cz

We study perturbations of the nonoscillatory half-linear differential equation

$$\left(r\left(t\right)\Phi\left(x'\right)\right)' + c\left(t\right)\Phi\left(x\right) = 0, \quad \Phi\left(x\right) := \left|x\right|^{p-2}x,\ p > 1.$$

We find explicit formulas for the functions $\hat{r}$, $\hat{c}$ such that the equation

$$\left[(r(t)+\lambda\hat{r}(t))\,\Phi\left(x'\right)\right]'+\left[c(t)+\mu\hat{c}(t)\right]\Phi(x)=0 \tag{$*$}$$

is conditionally oscillatory, i.e., there exists a constant γ such that $(*)$ is oscillatory if $\mu-\lambda>\gamma$ and nonoscillatory if $\mu-\lambda<\gamma$. The obtained results extend the previous results concerning two-parametric perturbations of the half-linear Euler differential equation.

The presented results were obtained jointly with Prof. Ondřej Došlý from Masaryk University, Brno.

Using the Lambert function in an exchange process with a time delay

Gamliel, Dan
Ariel University Center, Israel
E-mail: dang@ariel.ac.il

Spin exchange affects the lineshape in pulsed nuclear magnetic resonance (NMR). The evolution of the spin system with an exchange process is described by the Bloch–McConnell differential equations, in which it is assumed that each spin jumps instantaneously between two or more configurations. Using these equations one may calculate the observed lineshape under a variety of experimental settings. In this work the exchange process is generalized by assuming non-negligible time for the exchange jump. Two approximate descriptions for the system were proposed in a previous work. In one model the jump is assumed to occur via a transition state, which is treated in the standard equations as an ordinary state. In the other model, which is the focus of the present work, the jump is assumed to take place directly between the ordinary states, but with a time delay. The equations for a two-site system are:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}x_1(t)\\x_2(t)\end{pmatrix}+\begin{pmatrix}0&-k_{BA}\\-k_{AB}&0\end{pmatrix}\cdot\begin{pmatrix}x_1(t-\tau)\\x_2(t-\tau)\end{pmatrix}\qquad&\\+\begin{pmatrix}\frac{1}{T_{1A}}+k_{AB}&0\\0&\frac{1}{T_{1B}}+k_{BA}\end{pmatrix}\cdot\begin{pmatrix}x_1(t)\\x_2(t)\end{pmatrix}=\begin{pmatrix}u_1\\u_2\end{pmatrix}\end{aligned} \tag{1}$$

In this model the differential equations with a delay are solved using the complex Lambert function, defined by:

$$W(h)\cdot e^{W(h)}=h \tag{2}$$

In general this is a complex function with an infinite number of branches $W_p(h)$. Some properties of the solution modes are studied in this work, in particular the eigenvalues of the system and the stability of the solutions.

Unique periodic orbits of a DDE with piecewise linear feedback function

Garab, Ábel
University of Szeged, Hungary
E-mail: garab@math.u-szeged.hu

We study the delay differential equation $\dot{x}(t) = -ax(t) + bf(x(t-\tau))$ with feedback function $f(\xi) = \frac{1}{2}(|\xi+1| - |\xi-1|)$ and with real parameters $0 < a < |b|$ and $\tau > 0$. This equation is often applied in models of neural networks. We give necessary and sufficient conditions for existence and uniqueness of periodic orbits with prescribed oscillation frequencies (characterized by the values of a discrete Lyapunov functional). We also investigate the period function of the unique, slowly oscillating periodic solution (as a function of τ), which turns out to be important in examining periodic orbits of analogous systems of DDE's. This is a joint work with Prof. Tibor Krisztin.

On two aspects of multivalued dynamics

Garay, Barnabás M.
Pázmány Péter Catholic University, Hungary
E-mail: garay@math.bme.hu

I.) Stable/unstable manifolds We consider discrete-time and continuous-time hyperbolic dynamical system of the form

$$x_{k+1} \in Xx_k + f(x_k) + G(x_k), \quad \dot{x} \in Ax + f(x) + G(x)$$

where G is a well-behaved multivalued map. Persistence of the saddle structure is proved. The results are extended to systems with exponential dichotomy as well as to transversal homoclinics with Smale horseshoes.

II.) Finitely-many valued Poincaré mappings A standard piecewise affine planar model of hysteresis is reconsidered. Periodic orbits without self-intersection are studied and, in terms of the two real parameters, their full bifurcation analysis is given. Heteroclinic and homoclinic bifurcation curves responsible for larger and smaller supports of absolutely continuous invariant measures are also investigated. The main tool is a piecewise smooth, two-valued Poincaré mapping.

This is joint work with *Giovanni Colombo, Rudolf Csikja, Michal Fečkan*, and *János Tóth*.

Integro-differential equations with distributed delay or advanced

Goltser, Yakov

Ariel University Center, Israel

E-mail: jakovgoltser@014.net.il

Domosnitsky, Alexander

Ariel University Center, Israel

Our goal is to study the integro-differential equations in R^n with distributed delay or advanced

$$\frac{\mathrm{d}x}{\mathrm{d}t} = X\left(t, x, U, V\right), \tag{1}$$

where

$$U = \int_{-\infty}^{t} K\left(t, s\right) f\left(s, x\left(s\right)\right) \mathrm{d}s, \; V = \int_{t}^{\infty} K\left(t, s\right) f\left(s, x\left(s\right)\right) \mathrm{d}s, \tag{2}$$

and the kernel has the form

$$K\left(t, s\right) = \sum_{l=1}^{\infty} F_l\left(t\right) R_l\left(s\right).$$

We establish a connection between system IDE and countable system of ODE. Such a reduction allows use of results obtained earlier for the countable systems of ODE in study of integro-differential equations. For example, we will discuss problems related to applications of the truncated systems method (finite section) for investigation and solution of systems IDE (1),(2).

History-dependent decay rates for a logistic equation with infinite delay

Győri, István

University of Pannonia, Hungary

E-mail: gyori@almos.uni-pannon.hu

The logistic equation with infinite delay

$$\dot{N}\left(t\right) = N\left(t\right) \left\{ N - aN\left(t\right) - \int_{0}^{\infty} b\left(s\right) N\left(t-s\right) \mathrm{d}s \right\}, \quad t \geq 0,$$

with initial condition

$$N\left(t\right) = \phi\left(t\right), \quad t \leq 0,$$

is considered under conditions that force its solutions to approach a positive steady state at large times. It is demonstrated in this lecture that this rate of convergence depends on the initial history in some cases, and is independent of the history in others.

Our method is to show that $N(t)$ satisfies a linear Volterra integral equation. This equation is rescaled using appropriate weight functions. It is shown that the rescaled equation has asymptotically constant solutions by using admissibility properties of certain Volterra operators. For the kernels b and histories ϕ considered here, the appropriate weight functions involve subexponential functions.

It is a joint work with John A. D. Appleby and David W. Reynolds, School of Mathematical Sciences, Dublin City University, Dublin, Ireland.

Half-linear Euler differential equations in the critical case

Haladová, Hana

Masaryk University, Czech Rebublic

E-mail: haladovah@math.muni.cz

This is a joint work with Prof. Ondřej Došlý. We investigate oscillatory properties of the perturbed half-linear Euler differential equation

$$\left(\Phi\left(x'\right)\right)' + \frac{\gamma_p}{t^p}\Phi\left(x\right) = 0,\ \Phi\left(x\right) := \left|x\right|^{p-2} x,\ \gamma_p := \left(\frac{p-1}{p}\right)^p.$$

A perturbation is also allowed in the coefficient involving derivative.

Variational theory of hyperbolic Lagrangian coherent structures

Haller, George

McGill University, Canada

E-mail: george.haller@mcgill.ca

Lagrangian Coherent Structures (LCS) are dynamically evolving surfaces that govern the evolution of complex material patterns in moving fluids and solids. Examples of such patterns include oil spills, ash clouds, plankton populations, schools of fish and moving crowds. Because of their finite lifetime and aperiodic nature, LCS are not amenable to classical dynamical systems techniques involving asymptotic analysis or iterated maps. In this talk, I describe a mathematical theory that enables a rigorous extraction of LCS from observational flow data. In this approach, LCS are defined as invariant surfaces that extremize an appropriate finite-time normal repulsion or attraction measure in the extended phase space of the governing dynamical system. Solving this variational problem leads to computable sufficient and necessary criteria for LCS. I will show recent applications to large scale oceanic and atmospheric flow problems.

On second-order differentiability with respect to parameters for differential equations with state-dependent delays

Hartung, Ferenc
University of Pannonia, Hungary
E-mail: hartung.ferenc@uni-pannon.hu

In this talk we consider a class of differential equations with state-dependent delays. We show first and second-order differentiability of the solution with respect to parameters in a pointwise sense and also using the C-norm on the state-space, assuming that the state-dependent time lag function is piecewise strictly monotone.

Some classical results on the vibrating string revised

Hegedűs, Jenő
University of Szeged, Hungary
E-mail: hegedusj@math.u-szeged.hu

1) A slightly new understanding of *d'Alembert's formula* for the vibrations of the infinite strings will be presented with applications to the control theory and observation problems.
2) A sharpness of the well-known *Duhamel's principle* for the homogeneous Cauchy Problem, posed for the infinite string will be presented, too.

Minmax via Lyapunov

Hofbauer, Josef
University Vienna, Austria
E-mail: Josef.Hofbauer@univie.ac.at

I present some ways how to prove the minmax theorem of game theory via differential equations and Lyapunov functions.

Multidimensional stability of traveling waves in the discrete bistable equation

Hoffman, Aaron
Franklin W. Olin College of Engineering, USA
E-mail: aaron.hoffman@olin.edu

We establish multidimensional stability of planar fronts to the Allen–Cahn equation on $\mathbb{Z}^2$. The proof is based on comparison principles. The relevant sub- and super-solutions are adapted from recent work of Matano, Nara and Taniguchi on the Allen-Cahn equation in $\mathbb{R}^n$. The chief difficulty in passing to the discrete setting is the anisotropy of the discrete Laplacian. We have overcome this difficulty partially in that we restrict attention to fronts either facing the lattice directions or making a 45 degree angle with the lattice directions.

Uniqueness problems in inverse scattering and the spectral shift function

Horváth, Miklós
Budapest University of Technology and Economics, Hungary
E-mail: horvath@math.bme.hu

Consider the 3D inverse scattering problem with fixed energy, defined by the operator

$$\tau\left(\varphi\left(r\right)\right) = -\left(r_2\varphi'\right)'\left(r\right) + \left[r^2\left(q\left(r\right)-1\right)-1/4\right]\varphi\left(r\right), \quad r \in [0,\infty)$$

with some boundary condition at infinity. We present a uniqueness result for slowly decaying potentials, a characterization of the tail of the potential by the asymptotic behavior of the scattering amplitude, exact inequalities between the phase shifts and stability estimates for the reconstruction of the potential from noisy scattering data. The Krein spectral shift function corresponding to the operator is defined and uniqueness from the Krein function is verified together with some trace formulae.

Dynamics of differential equations with multiple state dependent delays

Humphries, Tony
McGill University, Canada
E-mail: tony.humphries@mcgill.ca

We demonstrate complex dynamics including bi-stability of periodic orbits, invariant tori, double Hopf bifurcations and period doubling in a very simple model problem consisting of a single scalar delay equation with two linearly state-dependent delays. We will use the model to illustrate the possible dynamics of state-dependent DDEs and the associated bifurcation structures.

Equations with delayed and advanced arguments in stick balancing models

Insperger, Tamás
Budapest University of Technology and Economics, Hungary
E-mail: inspi@mm.bme.hu

Wohlfart, Richárd
Budapest University of Technology and Economics, Hungary

Turi, János
University of Texas at Dallas, USA

Stépán, Gábor
Budapest University of Technology and Economics, Hungary

A stick balancing problem is considered, where the output for the feedback controller is provided by an accelerometer attached to the stick. This output is a linear combination of the stick's angular displacement and its angular acceleration. If the output is fed back in a PD controller with feedback delay, then the governing equation of motion is an advanced functional differential equation, since the third derivative of the angular displacement (the angular jerk) appears with a delayed argument through the derivative term. Equations with advanced arguments are typically non-causal and are unstable with infinitely many unstable poles. It is shown that the sampling of the controller may still stabilize the system in spite of its advanced nature. In the paper, different models for stick balancing are considered and discussed by analyzing the corresponding stability diagrams.

Existence and stability of periodic solutions of a state-dependent delay equation

Kennedy, Benjamin
Gettysburg College, USA
E-mail: bkennedy@gettysburg.edu

We discuss existence and stability of multiple periodic solutions of a scalar state-dependent delay equation

$$x'(t) = f\left(x\left(t - d\left(x_t\right)\right)\right)$$

where f is close, in a suitable sense, to a step function and $d\left(x_t\right)$ is given by a threshold condition

$$d\left(x_t\right) : \int_{t-d(x_t)}^{t} \theta\left(x\left(s\right)\right) \mathrm{d}s = 1.$$

Dynamic model of pandemic influenza: age structure, vaccination and long distance travel networks

Knipl, Diána
University of Szeged, Hungary
E-mail: knipl@math.u-szeged.hu

We present a 50 dimensional non-autonomous compartmental model with age structure and vaccination status and study the effect of age specific scheduling of vaccination during a pandemic influenza outbreak, when there is a race between the vaccination campaign and the dynamics of the pandemic. We evaluate and compare different scheduling strategies. The results show that age specific timing is paramount to vaccination planning. We show that the delay between developing immunity and vaccination is a key factor in evaluating the impact of vaccination campaigns. The applicability of our population dynamic model is demonstrated for the first wave of A(H1N1)v in Hungary. We also model the spread of influenza on long distance travel networks, which leads to a large system of delay differential equations.

References

[1] D. Knipl, G. Röst, Modelling the strategies for age specific vaccination scheduling during inluenza pandemic outbreaks, Math. Biosci. Eng. 8(1), 123-139 (2011)

Delayed difference equation exhibiting a huge number of stable periodic orbits

Kosovalic, Nemanja
York University, Canada
E-mail: nemanjak@mathstat.yorku.ca

We consider a nonlinear difference equation with a time delay which represents the simplest possible discretely updated neural network. That is, a self feedback system consisting of just a single neuron. We show that provided the time delay is large enough, this self feedback system exhibits a huge number of asymptotically stable periodic orbits. Therefore, such a network has potential for associative memory and pattern recognition.

The Cahn–Hilliard–Cook equation and its finite element approximation

Kovács, Mihály
University of Otago, New Zealand
E-mail: mkovacs@maths.otago.ac.nz

We study the nonlinear stochastic Cahn–Hilliard equation perturbed by additive colored noise given in the abstract form

$$\mathrm{d}X + (A^2X + Af(X))\mathrm{d}t = \mathrm{d}W, \quad t \in (0, T]; \quad X(0) = X_0,$$

where A denotes the Neumann Laplacian considered as an unbounded operator in the Hilbert space $H = L_2(\mathcal{D})$ and W is a Q-Wiener process in H with respect to a filtered probability space $(\Omega, \mathcal{F}, \mathbf{P}, \{\mathcal{F}_t\}_{t\geq 0})$. Here $\mathcal{D}$ is a bounded domain in $\mathbb{R}^d$, $d = 1, 2, 3$, and $f(s) = s^3 - s$. We discuss almost sure existence and regularity of solutions under the assumption $\|A^{1/2}Q^{1/2}\|_{\mathrm{HS}} < \infty$, where $\|\cdot\|_{\mathrm{HS}}$ denotes the Hilbert–Schmidt norm. In particular, we do not assume that A and Q has a common basis of eigenfunctions. We introduce spatial approximation by a standard finite element method and show how to obtain error estimates of optimal order on sets of probability arbitrarily close to 1. We also outline how these results can be used to prove mean-squared convergence, also referred to as strong convergence, without known rate. This is a joint work with Stig Larsson and Ali Mesforush, Chalmers University of Technology, Sweden.

Oscillation of the fourth-order nonlinear difference equations

Krejčová, Jana
Masaryk University, Czech Republic
E-mail: krejcovajana@mail.muni.cz

This is a joint work with Prof. Zuzana Došlá. We consider the nonlinear difference equation

$$\Delta\left(a_n\left(\Delta\left(b_n\left(\Delta\left(c_n\left(\Delta x_n\right)^{\gamma}\right)\right)^{\beta}\right)\right)^{\alpha}\right) + d_n x_{n+3}^{\delta} = 0,$$

where α, β, γ, δ are the ratios of odd positive integers and $\{a_n\}$, $\{b_n\}$, $\{c_n\}$ are positive real sequences defined for all $n \in \mathbb{N}$.

We shall classify the nonoscillatory solutions depending on the real sequence $\{d_n\}$.

For positive real sequence $\{d_n\}$ we establish necessary and sufficient conditions for equation to have nonoscillatory solutions with specific asymptotic behavior. We determine the sufficient conditions for equation to have the property A, that means there are only the oscillatory solutions of the equation.

For negative real sequence $\{d_n\}$ we investigate when the equation has the property B, that means that all nonoscillatory solutions are either Kneser solutions or strongly monotone solutions.

On asymptotic properties of differential equations with several power delays

Kundrát, Petr
Brno University of Technology, Czech Republic
E-mail: kundrat@fme.vutbr.cz

The contribution discusses the asymptotic bounds of solutions of differential equation with power coefficients and power delays in the form

$$\dot{y}(t) = \sum_{j=0}^{m} a_j t^{\alpha_j} y(t^{\lambda_j}),$$

where $\lambda_0 = 1$ and $0 < \lambda_i < 1$, $i = 1, \ldots, m$. Some additional assumptions on power coefficients are considered to obtain an asymptotic estimate for solutions of the studied differential equation. Results are illustrated by several examples.

On the order of operator splitting methods in reaction-diffusion equations

Ladics, Tamás
Szent István University, Hungary
tladics@math.bme.hu

To solve a problem in practice we use operator splitting and numerical schemes which we will call the *combined method*. The use of operator splitting as well as the numerical methods result in some error in the solution. The error generated purely by splitting is called *splitting error*. This is the difference of the exact solution and the approximate solution obtained by splitting (assumed that we know the exact solutions of the subproblems). Combined methods can generate both splitting error and *numerical error*. The study of this common effect on the solution is our main concern. Our aim is to characterize the error of this combined method therefore we symbolically calculate the order of the combined method for a *nonlinear* problem, the Fisher equation. We analyze the order of the error in the light of the characteristics of the splitting error and the *numerical error*. Numerical simulations and order estimation are performed, the results are in accordance with the symbolical calculations.

Hopf bifurcation for semiflows in Banach spaces

Lani-Wayda, Bernhard
University of Giessen, Germany
E-mail: Bernhard.Lani-Wayda@math.uni-giessen.de

We indicate how a Hopf bifurcation theorem for functional differential equations (FDE) can be proved correctly, following an approach that is described, but not carried out correctly in the literature. Then, using a different approach, we state and prove a Hopf bifurcation theorem for general semiflows in Banach spaces. The general statement includes the case of FDE.

Absolutely irreducible group actions and bifurcation

Lauterbach, Reiner
University of Hamburg, Germany
E-mail: lauterbach@math.uni-hamburg.de

One well known question in equivariant bifurcation is usually attributed to Ize and it reads, that in a one-parameter family of nonlinear maps which are equivariant with respect to an absolutely irreducible group actions a loss of stability implies bifurcation of equilibria.

One strategy to prove such a statement is to prove that absolutely irreducible group actions have at least one subgroup with an odd dimensional fixed point space.

We show that this assertion is false, by constructing absolutely irreducible group actions in $\mathbb{R}^4$ and $\mathbb{R}^8$ which no odd dimensional fixed point space.

We state a new conjecture on absolutely irreducible group actions and present some evidence.

We discuss bifurcation for most of these groups and note that there are interesting aspects concerning equivariant Hamiltonian systems.

Bubbles for delay differential equations with negative feedback

Liz, Eduardo
University of Vigo, Spain
E-mail: eliz@dma.uvigo.es

The family of delay differential equations

$$x'(t) = -\delta x(t) + f(x(t-\tau)), \tag{1}$$

is usually employed to model biological systems characterized by an instantaneous destruction rate δ, and a production function f which depends on the size of the relevant quantity x to be measured with a time delay τ. Sometimes, the destruction rate is a parameter that can be controlled (for example, by increasing or decreasing the harvesting or fishing effort in a population model); thus, it is important to study the influence of δ in the dynamics of equation (1).

In some models, it can be observed that there exist two values δ_1, δ_2 such that there is a locally stable equilibrium $k(\delta)$ for $\delta < \delta_1$ and $\delta > \delta_2$, while $k(\delta)$ is unstable for $\delta \in (\delta_1, \delta_2)$. A similar situation is usually referred to as *bubbling* in the framework of discrete dynamical systems, due to the bubble-like structures observed in the bifurcation diagram.

We introduce a definition of bubbles for a family of delay differential equations depending on a real parameter, and show an example in which a complete characterization of bubbles is possible. In other cases, the location of bubble-like structures can be estimated by finding lower and upper bounds for the amplitude of periodic solutions. We prove some results in this direction.

We apply our analytical results to the Nicholson's blowflies equation, and we show some numerical examples which allow us to figure out how an increasing mortality rate influences the dynamics, depending on the size of the delay. In particular, it is shown that increasing mortality can induce sustained oscillations in the population size, which is in agreement with some recent empirical studies.

This talk is based on a joint work with Tibor Krisztin, written within the framework of a Hungarian-Spanish Intergovernmental S&T Cooperation Programme, and partially supported by the Hungarian National Development Agency (TÁMOP-4.2.2/08/1/2008-0008 program).

Maximum principles and boundary value problem for neutral functional differential equations

Maghakyan, Abraham
Ariel University Center, Israel
E-mail: mabrham@walla.com

In this paper we obtain the maximum principles for the first order neutral functional differential equation

$$(Mx)(t) \equiv x'(t) - (Sx') - (Ax)(t) + (Bx)(t) = f(t), \quad t \in [0, \omega],$$

where $A : C_{[0,\omega]} \to L^\infty_{[0,\omega]}$, $B : C_{[0,\omega]} \to L^\infty_{[0,\omega]}$ are linear continuous positive operators, $S : L^\infty{}_{[0,\omega]} \to L^\infty_{[0,\omega]}$ is a linear operator, $C_{[0,\omega]}$ is the space of continuous functions and $L^\infty_{[0,\omega]}$ is the space of essentially bounded functions defined on $[0, \omega]$. New tests on positivity of the Cauchy function and its derivative are proposed. Results on existence and uniqueness of solutions for various boundary value problems are obtained on the basis of the maximum principles.

Some qualitative features of delay-differential equations with variable delays

Mallet-Paret, John
Brown University, USA
E-mail: jmp@dam.brown.edu

We examine several problems in delay-differential equations with variable delays. Among the new results are existence and asymptotics for multiple-delay problems; analyticity (or lack thereof) of solutions; and global bifurcation of periodic solutions. We shall also discuss some basic open questions arising in these contexts.

This is joint work with Roger Nussbaum.

Principal solutions of half-linear equation: yet another integral characterization

Mařík, Robert
Mendel University in Brno, Czech Republic
E-mail: marik@mendelu.cz

In this talk we study the principal solutions of nonoscillatory half-linear differential equation

$$(r(t)\Phi(x'))' + c(t)\Phi(x(t)) = 0,$$

where $\Phi(x) = |x|^{p-2}x$, $p > 1$. There are several known integral characterizations of principal solutions for this equation. However, an equivalent integral characterization which allows to deduce (non-)principality from one solution only is available only in the linear case $p = 2$. There are several known integral characterizations available for the general case $p > 1$, however, we either miss an equivalence in these characterizations or the knowledge of two independent solutions is required. In this contribution we provide an alternative integral characterization which extends some known results in this area.

On the asymptotic behavior of solutions of nonlinear differential equations of integer and also non-integer orders

Medveď, Milan
Comenius University, Slovakia
E-mail: Milan.Medved@fmph.uniba.sk

The first part of this talk is devoted to the equation

$$(\phi(u^{(n)}))' = g(t, u), \qquad t \geq t_0 \geq 1, \tag{1}$$

where $n \geq 1$, $\phi : R \to R$ is an increasing homeomorphism with $\phi(0) = 0$. Sufficient conditions for the existence of solutions of the equation (1) which are asymptotic at ∞ to a polynomial of degree $m \leq n-1$ will be presented. In the second part of the talk we present a new result on the asymptotic properties of the following fractional differential equation

$$D^{\alpha}x(t) = f(t, x(t)), \tag{2}$$

where

$$D^{\alpha}x(t) = \frac{1}{\Gamma(1-\alpha)} \frac{\mathrm{d}}{\mathrm{d}t} \int_0^t (t-s)^{-\alpha} x(s) \mathrm{d}s \tag{3}$$

is the Riemann–Liouville derivative of $x(t)$ of the non-integer order $\alpha \in (0,1)$.

Acknowledgment
The research was supported by the Slovak Grant Agency VEGA, *No.*1/0507/11.

A combinatorial framework for nonlinear dynamics

Mischaikow, Konstantin
Rutgers University, USA
E-mail: mischaik@math.rutgers.edu

Much of the focus of dynamical systems is on the existence and structure of invariant sets. What we have learned over the past century is that this existence and structure is extremely rich – for example, chaotic dynamics and/or bifurcations associate to nonuniformly hyperbolic systems. From the perspective of multiscale applications where one does not have exact models or parameters are poorly known detailed information about the dynamics can, in fact, be misleading.

With this in mind I will describe our efforts to develop a combinatorial and algebraic topological framework for nonlinear dynamics that is applicable to settings in which there are errors/limits to measurement and/or heuristic mathematical models. I will argue that this framework is computationally efficient and can provide mathematically rigorous descriptions of global dynamics of multidimensional systems.

I will demonstrate these ideas in the context of simple models from population biology.

Bifurcations of an ENSO-model

Nagy, Bálint
College of Dunaújváros, Hungary
E-mail: nagyb@mail.duf.hu

A three-variable model describing the El-Nino-Southern Oscillation is studied from the bifurcation point of view. Using the parametric representation method the bifurcation diagram is given.

A note on certain linear fractional difference equations

Nechvátal, Luděk

Brno University of Technology, Czech Republic

E-mail: nechvatal@fme.vutbr.cz

The contribution deals with an initial value problem for linear fractional difference equation in the form

$$\sum_{j=1}^{m} p_{m-j+1}(t)_0\nabla_h^{\alpha-j+1}y(t) + p_0(t)y(t) = 0, \quad t \in h\mathbb{Z},\ t \geq (m+1)h,$$

$$_0\nabla_h^{\alpha-j}y(t)\Big|_{t=mh} = y_{\alpha-j}, \quad j = 1, 2, \ldots, m,$$

where $\alpha \in \mathbb{R}^+$, $m \in \mathbb{Z}^+$ are such that $m-1 < \alpha \leq m$, $_0\nabla_h^{\gamma}$, $\gamma \in \mathbb{R}^+$, is a fractional backward h-difference and $y_{\alpha-j}$ are arbitrary real constants. Existence and uniqueness as well as the structure of the solution are discussed. Restricting to a simplified (two-term) equation, an explicit form of the solution via discrete Mittag–Leffler functions is presented.

Positive linear operators, tensor products and delay-differential equations

Nussbaum, Roger

Rutgers University, USA

E-mail: nussbaum@math.rutgers.edu

The results described in this lecture represent joint work with John Mallet-Paret of Brown University.

We shall discuss some intriguing connections among positive linear operators (in the sense of bounded linear operators L such that $L(K)$ is contained in K, where K denotes an appropriate cone of "nonnegative" elements of a Banach space), (injective) tensor products of Banach spaces and certain linear differential-delay equations. Consider the linear differential-delay equation

$$x'(t) = a(t)x(t) - b(t)x(t-1), \tag{$*$}$$

where $a(t)$ and $b(t)$ are given real-valued continuous functions defined for $t \geq 0$. Equation $(*)$ may arise by linearizing the nonlinear equation

$$x'(t) = f(x(t), x(t-1)) \tag{$**$}$$

about a given solution $y(t)$ of $(**)$, the case where $y(t)$ is periodic being of particular interest. Define $X := C([-1,0])$, the Banach space of continuous, real-valued functions g from $[-1,0]$ to the reals. If g is an element of X, there is a unique function $x(t;g)$ which solves $(*)$ for $t \geq 0$ and also satisfies $x(t;g) = g(t)$ for $-1 \leq t \leq 0$. For a fixed number

$c > 0$, define a bounded linear operator $A := U(c, 0) : X \longrightarrow X$ by $(A(g))(s) = x(c + s; g)$ for $-1 \leq s \leq 0$.

The m-fold injective tensor product of X with itself can be naturally identified with $C(M)$, the continuous real-valued functions on M, where M denotes the m-fold Cartesian product of $[-1, 0]$ with itself. Y_m, the m-fold exterior product of X, is naturally identified with a closed linear subspace of $C(M)$ as follows: If d is any permutation of $\{1, 2, \ldots, m\}$, $\mathrm{sgn}(d)$ is defined and $\mathrm{sgn}(d) = 1$ or $\mathrm{sgn}(d) = -1$. Any such permutation d naturally induces a map $d : M \longrightarrow M$. If f is an element of $C(M)$, f is an element of Y_m iff $f(d(x)) = \mathrm{sgn}(d)f(x)$ for all x in M and all permutations d. Y_m contains a closed cone K_m consisting of all functions f in Y_m such that $f(s_1, s_2, \ldots, s_m) \geq 0$ whenever $-1 \leq s_1 \leq s_2 \leq \cdots \leq s_m \leq 0$. If A is defined as above, A^m, the "m-fold exterior product of A with itself", is a natural bounded linear map of Y_m into itself. If $((-1)^m)b(t)$ is strictly positive for $0 \leq t \leq c$, we prove that A^m has positive spectral radius and maps K_m into itself. We also prove that A^m is u_0 positive in the sense of Krasnoselskii. When combined with known theorems concerning positive linear operators and the spectrum of tensor products of linear operators, our theorems yield new results even when $a(t)$ and $b(t)$ are constant.

Uniform persistence with applications to finite delay cellular neural networks

Obaya, Rafael
University of Valladolid, Spain
E-mail: rafoba@wmatem.eis.uva.es

We consider monotone skew-product semiflows generated by differential equations. We introduce some functions which determine the uniform persistence above a fixed minimal set K. We give some conditions of uniform persistence in terms of the upper Lyapunov exponent. These results are valid for cooperative ordinary, finite delay and reaction-diffusion systems. We apply the conclusions to some systems of delay equations describing cellular neural networks.

Some oscillatory properties of the second-order linear delayed differential equation

Opluštil, Zdeněk
Brno University of Technology, Czech Republic
E-mail: oplustil@fme.vutbr.cz

On the half-line $\mathbb{R}_+ = [0, +\infty[$ we consider the second-order linear delayed differential equation

$$u''(t) + p(t)u(\tau(t)) = 0 \tag{1}$$

where $p : \mathbb{R}_+ \to \mathbb{R}_+$ is a locally integrable function and $\tau : \mathbb{R}_+ \to \mathbb{R}_+$ is a continuous function such that $\tau(t) \le t$ for $t \ge 0$ and $\lim\limits_{t\to+\infty} \tau(t) = +\infty$. Some oscillation criteria of the equation (1) are established.

Global attractivity of a first order differential equation with applications to some mathematical models

Padhi, Seshadev
Birla Institute of Technology, India
E-mail: se_2312@yahoo.c.in

In this paper, we obtain a new sufficient condition for the global attractivity of solution of the delay differential equation $x'(t) + p(t)x(t - \tau) = 0$, $t \ge 0$ and $\tau > 0$ is constant. Further, the result has been applied to different mathematical models arising in ecology.

As an advantage, we have obtained some new sufficient conditions for the global attractivity of positive solutions of Hematopoiesis model

$$x'(t) = -a(t)x(t) + b(t)\frac{x^m(t-\tau(t))}{1 + x^n(t-\tau(t))},$$

Lasota–Wazewska model

$$x'(t) = -a(t)x(t) + b(t)e^{-\gamma(t)x(t-\tau(t))},$$

Nicholson's blowflies model

$$x'(t) = -a(t)x(t) + b(t)x^m(t-\tau(t))e^{-\gamma(t)x^n(t-\tau(t))},$$

the generalized logistic model of single species

$$x'(t) = x(t)\left[a(t) - b(t)x(t) - c(t)x(t-\tau(t))\right],$$

and some other models existing in ecology.

On two-point boundary value problems for two-dimensional nonlinear differential systems with strong singularities

Partsvania, Nino
A. Razmadze Mathematical Institute of I. Javakhishvili Tbilisi State University, Georgia
E-mail: ninopa@rmi.ge

We consider the differential system

$$u_1' = f_1(t, u_2) + f_{10}(t), \quad u_2' = f_2(t, u_1) + f_{20}(t) \tag{1}$$

with the two-point boundary conditions

$$u_1(a+) = 0, \quad u_1(b-) = 0, \tag{2_1}$$

or

$$u_1(a+) = 0, \quad u_2(b-) = 0. \tag{2_2}$$

Here $f_1 : [a,b] \times \mathbb{R} \to \mathbb{R}$, $f_{10} : [a,b] \to \mathbb{R}$, $f_2 :]a,b[\times \mathbb{R} \to \mathbb{R}$, and $f_{20} :]a,b[\to \mathbb{R}$ are continuous functions, and there is not excluded the case when the function f_2 with respect to the first argument has nonintegrable singularities at the points a and b such that

$$\int_a^{t_0} (t-a)|f(t,x)|\mathrm{d}t = \int_{t_0}^b (b-t)|f(t,x)|\mathrm{d}t = +\infty \text{ for } t_0 \in]a,b[, \ x \neq 0.$$

We are to find a solution of system (1) which along with (2_1) (along with (2_2)) satisfies the condition

$$\int_a^b u_2^2(t)\mathrm{d}t < +\infty. \tag{3}$$

In particular, the following Agarwal–Kiguradze type [1, 2] theorems are proved.

Theorem 1. *Let in the domain* $]a,b[\times\mathbb{R}$ *the inequalities*

$$\delta|x| \le f_1(t,x)\operatorname{sgn} x \le \ell_0|x|,$$

$$f_2(t,x)\operatorname{sgn} x \ge -\ell\left(\frac{1}{(t-a)^2} + \frac{1}{(b-t)^2}\right)|x|$$

be fulfilled, where δ, ℓ_0 *and* ℓ *are positive constants such that*

$$4\ell\ell_0 < \delta. \tag{4}$$

If, moreover,

$$\int_a^b (t-a)^{1/2}(b-t)^{1/2}\,|f_{20}(t)|\,\mathrm{d}t < +\infty, \tag{5}$$

then problem (1), (2_1), (3) *has at least one solution.*

Theorem 2. *Let* $f_i(t,0) \equiv 0$ $(i = 1,2)$, *and let in the domain* $]a,b[\times\mathbb{R}$ *the conditions*

$$\delta|x_1 - x_2| \le (f_1(t,x_1) - f_1(t,x_2))\operatorname{sgn}(x_1 - x_2) \le \ell_0|x_1 - x_2|,$$

$$(f_2(t,x_1) - f_2(t,x_2))\operatorname{sgn}(x_1 - x_2) \ge -\ell\left(\frac{1}{(t-a)^2} + \frac{1}{(b-t)^2}\right)|x_1 - x_2|$$

be fulfilled, where δ, ℓ_0 *and* ℓ *are positive constants, satisfying inequality* (4). *If, moreover, the function* f_{20} *satisfies condition* (5), *then problem* (1), (2_1), (3) *has one and only one solution.*

Analogous results are obtained for problem (1), (2_2), (3) as well.

Supported by the Shota Rustaveli National Science Foundation (Project # GNSF/ ST09_175_3-101).

References

[1] R. P. Agarwal and I. Kiguradze, Two-point boundary value problems for higher-order linear differential equations with strong singularities. *Boundary Value Problems* 2006, 1-32; Article ID 83910.

[2] I. Kiguradze, On two-point boundary value problems for higher order singular ordinary differential equations. *Mem. Differential Equations Math. Phys.* **32** (2004), 101-107.

Positive periodic solutions of differential equations with unbounded Green's kernel

Pati, Smita

Birla Institute of Technology, India

E-mail: spatimath@yahoo.com

Sufficient conditions have been obtained for the existence, nonexistence of at least two positive periodic solutions of a class of first order differential equation of the form

$$x'(t) = a(t)g(x(t))x(t) - \lambda b(t)f(x(h(t))),$$

which has an unbounded Green's function, where λ is a positive parameter, a, b and $h \in C(R, [0, \infty))$ are T-periodic functions, $\int_0^T a(t)\mathrm{d}t > 0$ and $\int_0^T b(t)\mathrm{d}t > 0$, $f : [0, \infty) \to [0, \infty)$ is continuous and $f(x) > 0$ for $x > 0$ and $g : [0, \infty) \to (0, \infty)$ is a continuous function.

Asymptotic behavior of the solutions of a linear difference equation with continuous arguments

Pituk, Mihály

University of Pannonia, Hungary

E-mail: pitukm@almos.uni-pannon.hu

Difference equations with continuous arguments may be viewed as special cases of neutral functional differential equations. In this talk, we give an asymptotic description of the solutions of a scalar linear difference equation with continuous arguments. The cases of commensurable delays and rationally independent delays will be discussed separately.

A Hamiltonian structure-preserving vorticity-dilatation formulation of the compressible Euler equations

Polner, Mónika
University of Szeged, Hungary
polner@math.u-szeged.hu

van der Vegt, J. J. W.
University of Twente, The Netherlands

Conservation laws and Hamiltonian structures are central to the mathematical study of fluid flow and have long been studied for the compressible Euler equations governing fluid flow in two and three dimensions. Preservation of the Hamiltonian structure and conservation laws after the discretization of the system is challenging and especially desirable in long time predictions, where conservation laws constrain the dynamics.

In this lecture we present a new Hamiltonian formulation of the compressible Euler equations. This formulation provides a mathematical framework for the development of numerical discretizations which can accurately compute vortical flows. Using a Hodge decomposition, the equations are transformed into a density-weighted vorticity and dilatation formulation, which can be shown to have a Hamiltonian structure. The associated Poisson bracket is used to find vorticity conservation laws.

Time delays, slope restrictions and monotonicity in nonlinear systems: almost linear behavior

Răsvan, Vladimir
University of Craiova, Romania
E-mail: vrasvan@automation.ucv.ro

By almost linear behavior of a nonlinear system it is understood the following gathering of qualitative properties: i) existence of a unique equilibrium (e.g. at the origin) – which is globally asymptotically stable – for the free system; ii) existence and global asymptotic stability of a unique limit regime of the forced system, the solution corresponding to the limit regime being "of the same type" as the forcing term; more precisely this forced solution is bounded for bounded forced term, being defined on the whole $\mathbb{R}$ axis and it is periodic if the forcing term is periodic and is almost periodic if the forcing term is such.

A most suited class of systems for the research on almost linear behavior is given by the class of systems with sector restricted nonlinearities and (possibly) linearly forced, described by

$$\dot{x} = Ax - b\phi(c^*x) + f(t) \tag{1}$$

with x and b being n-dimensional vectors, A a $n \times n$ matrix, $\phi : \mathbb{R} \mapsto \mathbb{R}$ a nonlinear function subject to the sector restrictions

$$\underline{\varphi} \leq \frac{\varphi(v)}{v} \leq \overline{\varphi},\ \phi(0) = 0 \tag{2}$$

and $f : \mathbb{R} \mapsto \mathbb{R}^n$ bounded on $\mathbb{R}$ and (possibly) periodic or almost periodic.

A. In the free (autonomous) case, when $f(t) \equiv 0$, there are known the absolute (i.e. global asymptotic and valid for all $\phi(\cdot)$ of the given class) stability criteria which may be expressed in terms of either a Liapunov function of the type "quadratic form plus integral of the nonlinear function" or a frequency domain inequality e.g. of Popov type. The two approaches are equivalent *via* the well known Yakubovich Kalman Popov (YKP) lemma [1].

An interesting research idea in the field has been to use more information on the function $\phi(\cdot)$. We discussed such a problem in a previous edition of this Colloquium [2], by considering the slope restricted nonlinear functions i.e. subject to (2) and to

$$-\underline{\alpha} \leq \phi'(v) \leq \overline{\alpha}. \tag{3}$$

The idea of incorporating only the slope restrictions in the frequency domain inequality was followed in that paper; an earlier criterion due to Yakubovich [3] incorporated both sector and slope restrictions.

We want to show in this paper that the Yakubovich proof technique, when properly used, allows some relaxation of the stability conditions given by the criterion of [2] incorporating only the slope restrictions about the nonlinearity.

B. In the forced case there exist two kinds of results. If only existence of a periodic solution is sought, then one can use the same conditions as in the stability problem (i.e. a Liapunov function of the type "quadratic form plus integral of the nonlinear function" or a Popov frequency domain inequality) to obtain dissipativeness in the sense of N. Levinson and further, existence of the periodic solution. Note that (2) is allowed to be fulfilled for "large deviations" only i.e. for $|v| > v_0$.

Introduction of (3) allows extending the Yakubovich dissipativeness criterion ensuring existence of the periodic solutions. A quite recent result [4] shows that such criteria (called in general *criteria with free parameters*) may give also global asymptotic stability of the periodic solution. Usually this stability, even its exponential (strongest) form is obtained, together with the existence of a periodic or almost periodic limit regime, by using a quadratic Liapunov function or, equivalently, a frequency domain inequality without free parameters.

C. The case of the systems with time delays is usually embedded in the more general case of an integral equation of the form

$$v(t) = \rho(t) - \int_0^t \kappa(t-\tau)\phi(v(\tau))\mathrm{d}\tau. \tag{4}$$

In this case the frequency domain approach is the only one that is applicable; the Popov type inequality is well known in this case [5], but its Yakubovich like extension may be obtained if only (3) is replaced by the global Lipschitz condition

$$0 \leq \frac{\phi(v_1) - \phi(v_2)}{v_1 - v_2} \leq L \tag{5}$$

if the forced systems are considered, existence and exponential stability of the forced oscillations is obtained by using a frequency domain inequality without free parameters. On the other hand the Lipschitz condition (5) should allow introduction of some free parameters at least in the periodic case, if a corresponding theorem of existence might be applied; the way suggested by [4] relies on the assumption that existence of the periodic solution is somehow ensured.

References

[1] Popov, V. M., Hyperstability of Control Systems, Springer Verlag, 1973.

[2] Răsvan, Vl., New results and applications of the frequency domain criteria to absolute stability of nonlinear systems, Proc. 3d Colloquium on Qualitative Theory of Differential Equations, August 1988, Szeged, Hungary, pp. 577-594, Colloq. Mathem. Societ. Janos Bolyai no. 53, North Holland, New York Amsterdam, 1990.

[3] Yakubovich, V. A., The method of the matrix inequalities in the the theory of stability for nonlinear controlled systems II. Absolute stability in the class of slope restricted nonlinearities (Russian), Avtom. i telemekh., 1965, 29, 4, pp. 577-590.

[4] Altshuller, D. A., Frequency domain criterion for stability of oscillations in a class of nonlinear feedback systems, Proc 3d IFAC Workshop Periodic Control Systems (PSYCO'07) Sankt Petersburg 2007 http://libphyscon.ru/?item=1396

[5] Corduneanu, C., Integral Equations and Stability of Feedback Systems, Academic Press, 1973.

Asymptotic theory of coupled nonlinear differential systems in the framework of regular variation

Řehák, Pavel

The Academy of Sciences of the Czech Republic

E-mail: rehak@math.cas.cz

(This is joint work with Serena Matucci, University of Florence, Italy.) We consider the nonlinear differential system

$$\left.\begin{aligned}(\Phi_\alpha(x'))' &= p(t)f(y),\\ (\Phi_\beta(y'))' &= q(t)g(x),\end{aligned}\right\} \tag{S}$$

where $\Phi_\lambda(u) = |u|^\lambda \operatorname{sgn} u$, α, β are positive constants, and p, q, f, g are certain continuous functions. We establish sufficient conditions guaranteeing the existence and regularly varying behavior of strongly decreasing solutions to (S). Strongly increasing solutions are solutions of whose both components and their derivatives tend to zero (as the independent variable tends to infinity). Study of such solutions in the framework of regular variation enables us to get much more precise information about their limit behavior. System (S) can be understood as of generalized Emden–Fowler type, and includes important classes of fourth order equations. As far as we know, this is the first time when the theory of regular variation was utilized in such higher order cases. To some extent, the results are new even in the scalar case, i.e., for nonlinear second order equations.

A Hartman type asymptotic formula for solutions of nonoscillatory half-linear differential equations

Řezníčková, Jana
Tomas Bata University in Zlín, Czech Republic
E-mail: reznickova@fai.utb.cz

In this contribution we present an asymptotic formula for nonoscillatory solutions of the half-linear differential equation

$$(r(t)\Phi(y'))' + c(t)\Phi(y) = 0, \quad \Phi(y) := |y|^{p-2}y,\ p > 1.$$

This equation is regarded as a perturbation of another nonoscillatory equation of the same form.

Some properties of solutions to parabolic partial differential equations with state-dependent delays

Rezounenko, Alexander
Kharkiv National University, Ukraine
E-mail: rezounenko@yahoo.com

The goal of the talk is to present and discuss results on the well-posedness of non-linear models described by parabolic partial differential equations with state-dependent delays. One of the approaches, based on an additional condition on the delay function introduced in [1] is developed. We propose and study a condition which is sufficient for the well-posedness of the corresponding initial value problem on the whole space of continuous functions C. The dynamical system is constructed in C and the existence of a compact global attractor is proved.

References

[1] A.V. Rezounenko, Differential equations with discrete state-dependent delay: uniqueness and well-posedness in the space of continuous functions, Nonlinear Analysis: Theory, Methods and Applications, 70 (11) (2009), 3978-3986.

On the solvability of some boundary value problems for linear functional differential equations

Rontó, András
Academy of Sciences of the Czech Republic
E-mail: ronto@ipm.cz

Conditions guaranteeing the solvability of certain three-point boundary value problems for a system of linear functional differential equations are obtained by using a special successive approximation scheme. We also establish some conditions necessary for a certain set belonging to the domain of the space variables to contain a point determining the initial value of the solution. An algorithm for selecting such points is also indicated.

Flow invariance for delay equations

Ruess, Wolfgang M.
Duisburg-Essen University, Germany
E-mail: wolfgang.ruess@uni-due.de

The object of the talk is the problem of flow-invariance for solutions to the non-autonomous partial differential delay equation of the form

$$\text{(PFDE)} \quad \begin{cases} \dot{x}(t) + B(t)x(t) \ni F(t; x_t), & t \geq 0 \\ x_{|I} = \varphi \in \hat{E}. \end{cases} \tag{1}$$

with $B(t) \subset X \times X$ $\omega-$accretive in a Banach space X, $I = [-R, 0]$, $R > 0$ (finite delay), or $I = \mathbb{R}^-$ (infinite delay), $\varphi : I \to X$ a given initial history out of a space E of functions from I to X, and $F(t; \cdot)$ a given history-responsive operator with domain $\hat{E}(t) \subset E$ and range in X.

Given closed subsets $\hat{E}(t)$ of E, and $\hat{X}(t)$ of X, $t \geq 0$, a subtangential condition is given ensuring that $(x_\varphi)_t \in \hat{E}(t)$, resp. $x_\varphi(t) \in \hat{X}(t)$ for $t \geq 0$, provided that $\varphi \in \hat{E}(0)$ (x_φ denoting the solution to (PFDE) for φ).

I shall also try to comment on the corresponding flow-invariance problem for state-dependent delay equations.

The result applies to diffusive population models with temporal or spatio-temporal averages over the past history, such as

$$\begin{cases} \dot{u}(t) - d(t)\Delta u(t) = a(t)u(t)\left[1 - b(t)u(t) - \int_{-1}^{0} u(t + r(s))\mathrm{d}\eta_t(s)\right], & t \geq 0 \\ u_{|[-R,0]} = \varphi \end{cases} \tag{2}$$

as well as corresponding models with the Laplacian being replaced by more general and nonlinear diffusion/absorption operators.

Global attractivity principle for functional differential equations

Röst, Gergely
University of Szeged / Hungarian Academy of Sciences, Hungary
E-mail: rost@math.u-szeged.hu

We present a general condition for global asymptotic stability in a wide class of FDEs, by ensuring that the deviation of the state of the solution from an equilibrium in the phase space norm is not increasing. In the special case of a single discrete delay, this reduces to a simple geometric condition. With this method we can easily reconstruct several classical global stability results and also give new results of absolute and delay dependent global attractivity for various applications in mathematical biology.

Navier–Stokes equations with Navier boundary conditions for an oceanic model

Sell, George
University of Minnesota, USA
E-mail: sell@math.umn.edu

We study this problem on a thin 3D domain, where one hopes to exploit the thinness ϵ to show that when ϵ is small:

1. the problem has a global attractor $A(\epsilon)$
2. there is a limiting problem when ϵ goes to 0, that
3. the limiting problem has a global attractor $A(0)$, and that
4. $A(0)$ is robust in the sense that $A(\epsilon)$ is semicontinuous in ϵ, as ϵ goes to 0.

We will describe the history of this problem in the context of fluid flows and the Navier–Stokes equations. We will see that the Navier boundary conditions (i.e., slip boundary conditions) are natural in addressing the physics of the problem. However, the Navier boundary introduce a new complexity in the analysis of the limiting problem.

We will introduce a new approach in the study of robustness which overcomes the added complexity of the problem.

Integrability and nonintegrability in terms of transcendental functions in dynamics

Shamolin, Maxim V.
Lomonosov Moscow State University, Russia
E-mail: shamolin@imec.msu.ru

The results of this scientific activity now are due to the study of the different problems of dynamics (in particular the rigid body motion in a resisting medium), where complete lists of transcendental first integrals expressed through a finite combination of elementary functions were obtained. This circumstance allowed the author to perform a complete analysis of all phase trajectories and highlight those properties of them which exhibit the roughness and preserve for systems of a more general form.

The complete integrability of those systems is related to symmetries of a latent type. Therefore, it is of interest to study sufficiently wide classes of dynamical systems having analogous latent symmetries. As is known, the concept of integrability is sufficiently broad and indeterminate in general. In its construction, it is necessary to take into account in what sense it is understood (it is meant that a certain criterion according to which one makes a conclusion that the structure of trajectories of the dynamical system considered is especially attractive and simple), in which function classes the first integrals are sought for. In my work, the author applies such an approach such that as first integrals, transcendental functions are elementary. Here, the transcendence is understood not in the sense of elementary functions (e.g., trigonometrical functions) but in the sense that they have essentially singular points (by the classification accepted in the theory of functions of one complex variable according to which a function has essentially singular points). In this case, it is necessary to continue them formally to the complex plane. As a rule, such systems are strongly nonconservative [1] and [2].

References

[1] M.V. Shamolin, *Methods of analysis of the dynamical systems with variable dissipation in Dynamics of a rigid body*, Ekzamen, Moscow, Russian Federation, 2007 [in Russian].

[2] M.V. Shamolin, *Dynamical systems with variable dissipation: approaches, methods, and applications*, Journal of Mathematical Sciences, **162:6** (2009), 741-908.

Periodic orbits in differential equations with state-dependent delay

Sieber, Jan
University of Portsmouth, UK
E-mail: Jan.Sieber@port.ac.uk

I show that periodic boundary-value problems for functional differential equations can be reduced to smooth low-dimensional algebraic systems of equations. The regularity assumptions on the right-hand side are identical to those stated in the review by Hartung *et al* (2006). In particular, they are set up such that the result can be applied to differential equations with state-dependent delays. For example, this reduction can be used to prove the Hopf bifurcation theorem for functional-differential equations with state-dependent delays. Thus, it provides an alternative to the original proof by Eichmann (2006). A preprint is available at http://arxiv.org/abs/1010.2391.

Nonlinear second order evolution equations with state-dependent delay

Simon, László
Eötvös Loránd University, Hungary
E-mail: simonl@ludens.elte.hu

We shall consider certain second order evolution equations where also the main part contains functional dependence and state-dependent delay on the unknown function. Existence and some qualitative properties of the solutions will be shown. The problem with state-dependent delay will be treated as a particular case of partial functional differential equations which were studied by applying the theory of monotone type operators to "abstract" evolution equations.

Oscillation theory of Sturm–Liouville equations with nonlinear dependence in spectral parameter

Šimon Hilscher, Roman
Masaryk University, Czech Republic
E-mail: hilscher@math.muni.cz

This talk is based on a joint work with Martin Bohner (Missouri University of Science and Technology) and Werner Kratz (University of Ulm). We consider the second order Sturm–Liouville eigenvalue problem

$$(r(t,\lambda)x')' + q(t,\lambda)x = 0, \quad t \in [a,b], \quad x(a) = 0 = x(b),$$

where r and q are given functions on $[a, b] \times \mathbb{R}$ which satisfy certain piecewise continuity and differentiability assumptions and they depend, in general, nonlinearly on the spectral parameter. The function $q(t, \cdot)$ is nondecreasing in λ (which is weaker than the strict monotonicity of q used in the current literature) and the function $r(t, \cdot)$ is positive and nonincreasing in λ on $\mathbb{R}$ for every $t \in [a, b]$. We introduce the notion of a finite eigenvalue and prove the oscillation theorem relating the number of finite eigenvalues which are less than or equal to a given value of λ with the number of proper focal points (generalized zeros) of the principal solution of the equation in $(a, b]$. We also define the corresponding geometric multiplicity of finite eigenvalues in terms of finite eigenfunctions and show that the algebraic and geometric multiplicities coincide. Under our assumptions, the finite eigenvalues are real, isolated, and bounded from below. Thus, the traditional results for the classical Sturm–Liouville eigenvalue problem are generalized from the case of $r(t, \lambda) \equiv r_0(t) > 0$ and $q(t, \lambda) = q_0(t) - \lambda$ to the general nonlinear dependence on λ. Our results follow from a more general theory of linear Hamiltonian differential systems with nonlinear dependence on the spectral parameter and with Dirichlet boundary conditions.

References

[1] M. Bohner, W. Kratz, R. Šimon Hilscher, Oscillation and spectral theory for linear Hamiltonian systems with nonlinear dependence on the spectral parameter, submitted (2011).

On the parameter depedence of the resolvent function of functional differential equations

Slezák, Bernát

University of Pannonia, Hungary

E-mail: slezakb@almos.vein.hu

Abstract state-dependent functional differential equations type of

$$x'(t) = f\left(t, x_t, x\left(t - \tau\left(t, x_t, \theta\right)\right), \omega\right), \quad x_\sigma = \phi$$

are investigated. The continuous and smooth parameter-dependence of the resolvent function are proved, including the initial time-parameter σ. Not only in the special case of the state independent functional differential equations type of

$$x'(t) = f\left(t, x_t, \omega\right), \quad x_\sigma = \phi,$$

but also for the finite dimensional ordinary differential equations stronger results are obtained then the classical ones.

Coexistence of limit cycles and invariant conic in a class of polynomial systems on the plane

Szántó, Iván

Universidad Federico Santa Maria, Chile

E-mail: ivan.szanto@usm.cl

In this paper for a certain class of polynomial systems with a nondegenerate invariant conic, we show that the invariant conic can coexist with a center or limit cycles. The obtained limit cycles can bifurcate out of a non hyperbolic focus at the origin and can bifurcate out of a perturbed Hamiltonian system.

On a SEIR epidemic model with delay

Székely, László

Szent István University, Hungary

E-mail: szekely.laszlo@gek.szie.hu

We consider an SEIR-type disease transmission model with fixed latency period, standard incidence and variable population size. Infected individuals are assumed to be unable to give birth, and individuals recover from the disease and acquire permanent immunity with probability f, and dies from the disease with probability $1-f$. Two threshold parameters are found which determine whether the disease dies out or remains endemic and whether the size of the population tends to zero, remains finite or grows exponentially. In addition, for the proportional form of the model we give a complete classification of the equilibria by a novel application of the envelope method.

Joint work with Gergely Röst, Shao Yuan Huang and Róbert Vajda.

Observation problems posed for the Klein–Gordon equation

Szijártó, András

University of Szeged, Hungary

E-mail: szijarto@math.u-szeged.hu

Transversal vibrations $u = u(x,t)$ of a string of length l with fixed ends are considered, where

$$u_{tt}(x,t) = a^2 u_{xx}(x,t) + cu(x,t), \qquad (x,t) \in [0,l] \times [0,T], \quad a > 0,\ c < 0.$$

Sufficient conditions are obtained that guarantee the solvability of observation problems with given state functions $f_1(x),\ f_2(x),\ x \in [0,l]$ at two distinct time instants $0 < t_1 < t_2 \leq T$. The essential conditions are the following: classical or generalized H^s smoothness

of $f_1,\ f_2$ and representability of $t_2 - t_1$ in terms of rational multiples of $\dfrac{2l}{a}$. The reconstruction of the unknown initial data $u(x,0)$ and $u_t(x,0)$ are given by means of their Fourier expansions. Both classical and generalized solutions are considered.

On the convergence of solutions of nonautonomous functional differential equations

Terjéki, József
University of Szeged, Hungary
E-mail: terjeki@math.u-szeged.hu

Bartha, Mária
University of Szeged, Hungary

We investigate the equation

$$x'(t) = -h(x(t)) + g\left(t, x_t\right) \tag{P}$$

where $h : \mathbb{R} \to \mathbb{R}$, $g : \mathbb{R} \times C \to \mathbb{R}$ are continuous functions, $0 < r < \infty$, $C_r = \{\phi : [-r,\ 0] \to \mathbb{R},\ \phi \text{ is continuous}\}$. We suppose that

(1) h is increasing,

(2) $g(t,\ \phi) \le h\left(\max\{\phi(s) : s \in [-r,\ 0]\}\right) \quad \text{for all} \quad \phi \in C_r,\ t \ge 0$,

(3) the initial value problem $u'(t) = -h(u(t)) + h(\alpha)$, $u(0) = \alpha$ has a unique solution $u(t) = \alpha$.

Among others we prove if conditions (1), (2), (3) are valid, $x(t)$ is a solution of (P) on $[0, \infty)$, then $\lim_{t\to\infty} x(t)$ exists or $\lim_{t\to\infty} x(t) = -\infty$.

On the integrability of non-autonomous differential equations of second order with variable deviating argument

Tunç, Cemil
Yüzüncü Yıl University, Turkey
E-mail: cemtunc@yahoo.com

This paper considers a non-autonomous differential equation of second order with multiple variable deviating arguments. By using time-varying delay Lyapunov–Krasovskii functional, we propose to find some sufficient conditions for the solutions of the equation considered are quadratic integrable. An example is given to show the applicability of our result.

A class of nonlinear differential equations on ordered Banach spaces

Ungureanu, Viorica Mariela
Constantin Brâncuşi University, Romania
E-mail: vio@utgjiu.ro

In this talk we discuss a class of time-varying nonlinear differential equations defined on infinite dimensional ordered Banach spaces, which includes as special cases many of the differential Riccati equations arising in stochastic optimal control problems. Using a linear matrix inequalities (LMI) approach and the properties of the evolution operators of linear differential equations generating a positive evolution, we investigate the problem of existence of certain global solutions for these differential nonlinear equations. So, we derive necessary and sufficient conditions for the existence of maximal, stabilizing or minimal solutions for the studied class of differential equations.

Periodic orbits for delay differential equations

Vas, Gabriella
University of Szeged / Hungarian Academy of Sciences, Hungary
E-mail: vasg@math.u-szeged.hu

The talk considers the scalar delay differential equation

$$\dot{x}(t) = -\mu x(t) + f(x(t-1)),$$

where $\mu > 0$ and $f : \mathbb{R} \to \mathbb{R}$ is a monotone nonlinear map. Step functions and continuously differentiable functions are studied as well.

Step feedback functions are easy to handle: the above equation with a step function f can be reduced to ordinary differential equations, hence specific infinite dimensional problems related to the equation (e. g. the construction of periodic orbits) can be simplified to finite dimensional ones.

The goal of the present talk is to show that the existence of periodic orbits for smooth nonlinear maps can be proved by considering step feedback functions first, and then by using perturbation theorems. A key technical property in the carry-over procedure is the hyperbolicity of periodic orbits in question. This feature can be verified in a straightforward way for equations with step functions, and the perturbation techniques preserve the hyperbolicity for smooth nonlinearities. We remark that confirming hyperbolicity of periodic orbits is still an infinite dimensional problem, which is solved only in some particular cases like our one.

This is a joint work with Tibor Krisztin.

References

[1] Krisztin, T., Vas, G., Large-amplitude periodic solutions for a differential equation with delayed positive feedback, accepted by *Journal of Dynamics and Differential Equations.*

[2] Vas, G., Infinite number of stable periodic solutions for an equation with negative feedback, *E. J. Qualitative Theory of Diff. Equ.* 18. (2011), 1-20.

Almost periodic skew-Hermitian and skew-symmetric linear differential systems

Veselý, Michal

Masaryk University, Czech Republic

E-mail: michal.vesely@mail.muni.cz

One modifiable method for constructing almost periodic functions is presented in connection with almost periodic skew-Hermitian and skew-symmetric homogeneous linear differential systems. Applying this method, it is proved that in any neighbourhood of a skew-Hermitian or skew-symmetric system there exists an almost periodic system which does not possess a nontrivial almost periodic solution.

Rapid variation on time scales with applications to dynamic equations

Vítovec, Jiří

University of Technology Brno, Czech Republic

E-mail: vitovec@feec.vutbr.cz

We discuss a rapid variation on time scales, which extends well studied classic theory of continuous and discrete cases. We introduce two concepts of definitions of rapidly varying functions and then show the relation between them. As an application, we consider the second order half-linear dynamic equation

$$\left(\Phi\left(y^{\triangle}\right)\right)^{\triangle} - p(t)\Phi\left(y^{\sigma}\right) = 0$$

on time scales and give necessary and sufficient conditions under which certain solutions of this equation are rapidly varying. The talk is finished by a few interesting open problems and concluding comments of this themes.

References

[1] J. Vítovec, Theory of rapid variation on time scales with applications to dynamic equations, *Arch. Math.* (Brno) **46** (2010), 263-284.

[2] P. Řehák, J. Vítovec, q-Karamata functions and second order q-difference equations, *Electronic Journal of Qualitative Theory of Differential Equations*, accepted (2011).

On linearization for neutral equations with state-dependent delays

Walther, Hans-Otto
University of Giessen, Germany
E-mail: Hans-Otto.Walther@math.uni-giessen.de

Neutral functional differential equations of the form

$$x'(t) = g(\partial x_t, x_t)$$

define continuous semiflows G on closed subsets in manifolds of C^2-functions, under hypotheses designed for the application to equations with state-dependent delay. Differentiability of the solution operators $G(t, \cdot)$ in the usual sense is not available, but for a certain variational equation along flowlines the initial value problem is well-posed. Using this variational equation we prove a principle of linearized stability which covers the prototype

$$x'(t) = A(x'(t + d(x(t)))) + f(x(t + r(x(t))))$$

with nonlinear real functions A, $d < 0$, f, $r \leq 0$. Special cases of the latter describe the interaction of two kinds of behaviour, namely, following a trend versus negative feedback with respect to a stationary state.

Spreading speeds and traveling waves for non-cooperative reaction-diffusion systems

Wang, Haiyan
Arizona State University, USA
E-mail: wangh@asu.edu

In this talk, we shall establish the spreading speed for a large class of non-cooperative reaction-diffusion systems and characterize the spreading speed as the slowest speed of a family of traveling wave solutions. These results are based on several recent results on spreading speeds for cooperative reaction-diffusion systems. The results are applied to a non-cooperative system describing interactions between ungulates and grass. We shall identify conditions on the parameters and the nonlinearity of the model under which a population of ungulates can invade an infinite grassland.

Global continuation of periodic solutions of differential systems with state-dependent delay

Wu, Jianhong
York University, Canada
E-mail: wujh@mathstat.yorku.ca

The talk will start with a brief introduction how state-dependent delay arises naturally from neural signal processing, and how delay adaptation plays a critical role in a newly developed subspace clustering (pattern recognition) algorithm. The talk will then focus on the application of equivalent-degree approach to examine the birth and global continuation of Hopf bifurcations of periodic solutions for such a class of state-dependent system. Some recently developed techniques will be described to show how local bounds of periods of periodic solutions can be glued together globally along a continuation of periodic solutions in the Fuller space, and how the global bifurcation theory can be applied to establish the co-existence of slowly and rapidly oscillatory periodic solutions when the parameter is away from its critical values where local Hopf bifurcations take place. Results on the analyticity of bounded solutions and how these results are used in establishing global continuation of periodic solutions will also be discussed.

New results in theory of Friedrichs extension

Zemánek, Petr
Masaryk University, Czech Republic
E-mail: zemanek@math.muni.cz

We present an overview of new results obtained by the author in the theory of the Friedrichs extension. The concept of the Friedrichs extension was introduced in [1], where it was shown that for a symmetric, densely defined, linear operator $\mathcal{L}$ which is bounded below in a Hilbert space H, there exists a self-adjoint extension of $\mathcal{L}$ with the same lower bound. The topic has been intensively studied, especially for the second order Sturm–Liouville differential equations. These results were generalized by the author for operators associated with the second order Sturm–Liouville equations on time scales, see [3], for the operators associated with the linear Hamiltonian differential systems, see [2], and most recently for operators associated with the higher-order Sturm–Liouville equations on time scales in the form

$$\sum_{i=0}^{n} (-1)^{n-i} \left(p_i y^{\triangle^{n-i-1}\nabla}\right)^{\nabla^{n-i-1}\triangle}(t) = 0, \quad t \in \mathbb{T}.$$

References

[1] K. Friedrichs, *Spektraltheorie halbbeschränkter Operatoren und Anwendung auf die Spektralzerlegung von Differentialoperatoren, zweiter Teil* (in German), Math. Ann. **109** (1934), no. 1, 685-713.

[2] R. Šimon Hilscher and P. Zemánek, *Friedrichs extension of operators defined by linear Hamiltonian systems on unbounded interval*, in "Equadiff 12", Proceedings of the Conference on Differential Equations and their Applications (Brno, 2009), J. Diblík, O. Došlý, P. Drábek, and E. Feistauer, editors, Math. Bohem. **135** (2010), no. 2, 209-222.

[3] P. Zemánek, *Krein-von Neumann and Friedrichs extensions for second order operators on time scales*, in "Dynamic Equations on Time Scales and Applications" (L. H. Erbe and A. C. Peterson, editors), Int. J. Dyn. Syst. Differ. Equ. **3** (2011), no. 1-2, 132-144.

Global dynamics of a delay differential equation with spatial non-locality in an unbounded domain

Zou, Xingfu
University of Western Ontario, Canada
E-mail: xzou@uwo.ca

In this talk, I will report some results on the global dynamics of a class of differential equations with temporal delay and spatial non-locality in an unbounded domain, obtained in a recent joint work with Dr. Taishan Yi. Such an equation can be used to model the population dynamics for those species whose immature individuals diffuse but mature individuals do not (e.g. barnacles and mussels). In order to overcome the difficulty due to non-compactness of the spatial domain, we introduce the compact open topology and describe the delicate asymptotic properties of the nonlocal delayed effect. Then, we establish some *a priori* estimate for nontrivial solutions which enables us to show the permanence of the equation. Combining these results with a dynamical systems approach, we are able to determine the global dynamics of the equation under appropriate conditions. Applying the main results to particular models with the Ricker's birth function and Mackey–Glass's hematopoiesis function, we obtain threshold results for the global dynamics of these two models. We explain why our results on the global attractivity of the positive equilibrium in $C_+ \setminus \{0\}$ under the *compact open topology* will become invalid in $C_+ \setminus \{0\}$ with respect to the *usual suprermum norm*, but the positive equilibrium can remain attractive in a *subset* of $C_+ \setminus \{0\}$ with respect to the supremum norm.

Dictionary

Pronunciation guide

a	similar to **o** in *hot*
á	like **u** in *hut*, but twice as long
e	like **e** in *pen*
é	like **a** in *play*
i	similar to **i** in *sit*
í	like **ee** in *meet*
o	like **o** in *force*
ó	like **aw** in *paw*
ö	like German **ö** or **e** in French *le*
ő	like **ö**, only longer
u	like **u** in *put*
ú	like **oo** in *fool*
ü	like German **ü** or **u** in French *tu*
ű	like **ü**, only longer
c	like **ts** in *hats*
cs	like **ch** in *teacher*
dz	like **ds** in *roads*
dzs	like **j** in *jam*
g	like **g** in *good*
gy	similar to **d** in *during*
j/ly	like **y** in *you*
ny	like **n** in *new*
s	like **sh** in *ship*
sz	like **s** in *sea*
ty	similar to **t** in *student*
zs	like **s** in *pleasure*

The consonants not listed above are pronounced the same way as in English.

Greetings, basic phrases

Good morning!	Jó reggelt!	Thank you!	Köszönöm!
Good afternoon!	Jó napot!	Excuse me!	Elnézést!
Good evening!	Jó estét!	yes	igen
Good night!	Jó éjszakát!	no	nem
Goodbye!	Viszontlátásra!	differential equation	differenciálegyenlet

Getting around, shopping

Where is the ...?	Merre van a ...?	bank	bank
street	utca	railway station	vasútállomás
square	tér	post office	posta
to the left	balra	department store	áruház
to the right	jobbra	How much is this?	Mennyibe kerül?
straight	egyenesen		

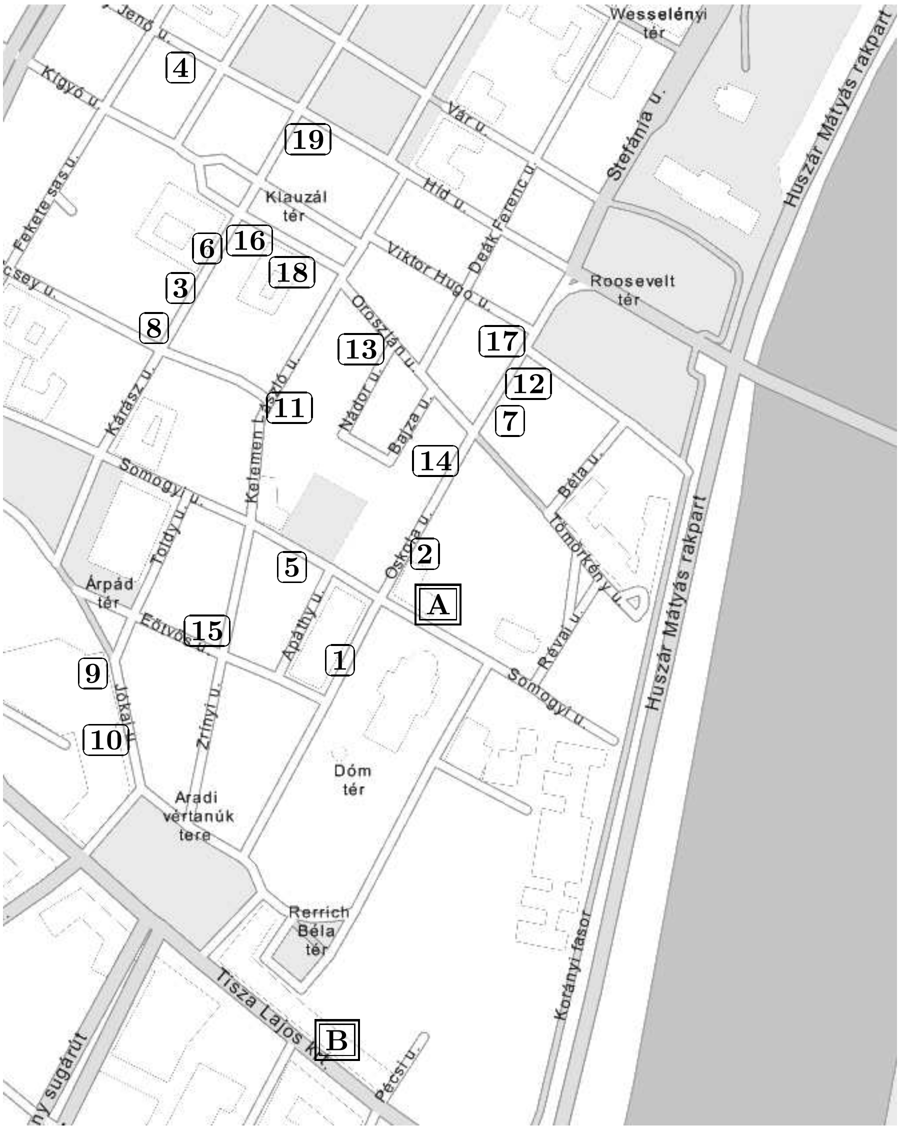
Numbers

1	egy	30	harminc
2	kettő	31	harmincegy
3	három	32	harminckettő
4	négy	40	negyven
5	öt	50	ötven
6	hat	60	hatvan
7	hét	70	hetven
8	nyolc	80	nyolcvan
9	kilenc	90	kilencven
10	tíz	100	száz
11	tizenegy	200	kettőszáz
12	tizenkettő	300	háromszáz
20	húsz	1000	ezer
21	huszonegy	2000	kétezer
22	huszonkettő	3000	háromezer

Meals

breakfast	reggeli	beer	sör
lunch	ebéd	wine	bor
dinner	vacsora	juice	gyümölcslé
tea	tea	apple	alma
coffee	kávé	orange	narancs
bread	kenyér	peach	barack
egg	tojás	sour cherry	meggy
butter	vaj	pear	körte
milk	tej	cake	sütemény
cheese	sajt	ice cream	fagylalt
cream	tejszín	salt	só
soup	leves	pepper	bors
chicken	csirke	sugar	cukor
turkey	pulyka	potato	krumpli
fish	hal	rice	rizs
beef	marha	vegetable	zöldség
pork	sertés	fruit	gyümölcs
lamb	bárány	cold	hideg
water	víz	hot	meleg

Map of downtown



A *Hungarian Academy of Sciences (Conference site)*

B *Internet access*

There is a computer lab with Internet access on the first floor of the Bolyai Building. Please enter the building through the door opening to Tisza Lajos körút, go upstairs to the first floor, turn left and look for computer lab Szőkefalvi (number 153).

Opening hours:

Tuesday 13:00-19:00
Wednesday 13:00-19:00
Thursday 13:00-19:00
Friday 8:00-14:00

In addition, there is a free WiFi access to Internet in the Conference site; the password is "Academy1961".

Where to eat

Fast food restaurants

1 *Fesztivál Restaurant*
A self-service restaurant offering soups and main dishes.

2 *Hamm-burger büfé*
A wide variety of hamburgers for a low price.

3 *Gülüm Döner Kebab*
A Turkish fast food restaurant inside Átrium Courtyard.

4 *Chili Grill*
A small shop serving delicious salads, hamburgers and wraps.

5 *Boci Tejivó*
A non-stop and cheap fast-food restaurant (so called milk bar) serving omlettes, dumplings and pancakes.

6 *Grafitto Pizzéria*
An Italian restaurant a bit on the expensive side, serving salads, pizzas and pastas.

7 *Pizzaguru*
A pizzeria also offering soups and gyros.

8 *McDonald's*

9 *Burger King*

10 *Wok n' Go*
A Chinese fast food restaurant.

Restaurants

11 *Brnoi Étterem és Söröző*
Restaurant and pub. Moderately priced daily menu is available.

12 *Vendéglő a régi hídhoz*
A traditional Hungarian restaurant.

13 *John Bull Pub*
A British-style restaurant and pub.

14 *Alabárdos Étterem*
A nice restaurant also serving daily menu.

15 *Pagoda Kínai Étterem*
A Chinese restaurant.

In restaurants it is customary to give 5-10% extra as a tip unless you are unhappy with the food or the service. You are not expected to give a tip in fast food restaurants.

Confectionaries

16 *A Cappella*
Szeged's most popular ice cream shop.

17 *Palánk*
Confectionary with smaller selection of ice creams and cakes.

18 *Hatos rétes*
Offers strudels and pancakes.

19 *Kerek Perec*
Bakery.