ABSTRACTS

Functional differential equations with piecewise constant argument

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We introduce a new class of functional differential equations with functional response on piecewise constant argument. It contains functional differential equations with continuous time as well as differential equations with piecewise constant argument. In this talk, we concentrate only on retarded equations, but one can easily extend the discussion to any type of piecewise constant argument and functional differential equations. At the end of the report, we suggest how one can apply the systems for solution of real world problems, provided more general systems for future investigations.

Preservation of asymptotic behaviour when discretising highly nonlinear stochastic differential equations

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This talk gives a survey of recent work concerning the asymptotic behaviour of discretised stochastic differential equations. Preservation of stability, boundedness, unboundedness, positivity as well as almost sure convergence, growth and explosion rates, becomes delicate in situations in which drift or diffusion coefficients have unbounded derivatives in regions of the state space to which the solution of the underlying SDE ultimately tends. We briefly discuss the merits of explicit and implicit methods of discretisation, deterministic but variable time stepping for equations with state-independent noise, and state-dependent time stepping for equations with state-dependent noise.

On the lumpability of differential equations

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This talk deals with exact lumpability of dynamical systems, namely, the possibility of projecting the dynamics onto a smaller state space in which a self-contained dynamical description exists. The process is also called lumping, aggregation, or reduction in various contexts. As a first setting we consider systems whose evolution is described by bounded or unbounded linear operators on general Banach spaces and obtain conditions for lumpability using semigroup theory. We also present the problem from a dual space point of view in the language of sun dual spaces. In the second setting we consider nonlinear systems on finite-dimensional manifolds. We give several examples for applications.

Solutions of two-point BVP via phase-plane analysis

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We consider equation of the type

$$x'' + f(x)x'^2 + g(x) = 0 \tag{1}$$

together with Neumann boundary conditions

$$x'(a) = 0, \qquad x'(b) = 0. \tag{2}$$

Transforming (1) to conservative form

$$u'' + h(u) = 0, \tag{3}$$

where $h(u) = g(x(u))e^{F(x(u))}$, we obtain the existence and multiplicity results for the problem (1), (2).

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Periodic solutions of a differential equation with a queueing delay

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We consider a differential equation with a state-dependent delay motivated by a queueing process. The time delay is determined by an algebraic equation involving the length of the queue. For the length of the queue a discontinuous differential equation holds. We formulate an appropriate framework to study the problem, and show that the solutions define a Lipschitz continuous semiflow in the phase space. The main result guarantees the existence of slowly oscillating periodic solutions.

Immuno-epidemiology of a population structured by immune status

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When the body gets infected by a pathogen the immune system develops pathogen-specific immunity. Induced immunity decays in time and years after recovery the host might become susceptible again. Exposure to the pathogen in the environment boosts the immune system thus prolonging the time in which a recovered individual is immune. Such an interplay of within host processes and population dynamics poses significant challenges in rigorous mathematical modeling of immuno-epidemiology. We propose a framework to model SIRS dynamics, monitoring the immune status of individuals and including both waning immunity and immune system boosting. Our model is formulated as a system of two ordinary differential equations (ODEs) coupled with a PDE. After showing existence and uniqueness of a classical solution, we investigate the local and the global asymptotic stability of the unique disease-free stationary solution. Under particular assumptions on the general model, we can recover known examples such as large systems of ODEs for SIRWS dynamics, as well as SIRS with constant delay. This is a joint work with G. Röst.

Bounds for the expected value of one-step processes

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We consider continuous-time Markov processes with finite discrete state-space. Epidemic propagation is an important example of such processes. When the state-space is large, mean-field models are used to approximate the process. We derive explicit bounds on the expected value of the process, bracketing it between the mean-field model and another simple ODE. Our approach is based on standard comparison theorems of ODEs which makes it quite elementary compared to the existing proofs in this topic. We illustrate our results on the SIS epidemic process and the voter model. Joint work with B. Armbruster and P. L. Simon.

On the soultions to Navier–Stokes problem for some non-Newtonian cases

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The problem considered here is the steady boundary layer flow due to a moving flat surface in an otherwise quiescent non-Newtonian fluid medium moving at a speed of $U_w(x)$. The laminar boundary layer equations expressing conservation of mass and the momentum boundary layer equations for an incompressible fluid are written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \qquad u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2},$$

where x and y denote the Cartesian coordinates along the sheet and normal to the sheet, respectively; while u and v are the velocity components of the fluid in the x and y directions, ν stands for the kinematic viscosity. We consider the boundary-layer flow induced by a continuous surface stretching with velocity $U_w(x) = Ax^m$, where m is a positive integer. The corresponding boundary conditions on the surface are $u(x,0) = U_w(x)$ and $v(x,0) = 0$, far from the surface $\lim_{y\to\infty} u(x,y) = 0$. The boundary layer equations are solved in similarity form via the Lyapunov energy method. We show that this problem has positive global solutions.

Herd immunity caused by a toxoid vaccine – the case study of diphtheria by dynamic models

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Herd immunity is the indirect protection of a population from an infectious disease when the proportion of immune individuals is sufficiently high. Vaccination is a very effective way of achieving herd immunity. However, there are some vaccines, so-called toxoid vaccines, which do not protect against the pathogen, but neutralize the toxins created by the pathogen, thus preventing serious disease. It is a puzzling observation that even toxoid vaccines can generate herd immunity. We consider the example of diphtheria, and explain this phenomenon by a mathematical model. We also fit our model to the times series of diphtheria in Romania after the introduction of the vaccine. This is a joint work with Gergely Röst.

Long time behaviour of delay 2D Navier–Stokes equations

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In this talk we will show several methods to analyze the long time behaviour of solutions to 2D Navier–Stokes models when some hereditary characteristics (constant, distributed or variable delay, memory, etc.) appear in the formulation. First the local stability analysis of steady-state solutions is studied by using several methods: the theory of Lyapunov functions, the Razumikhin–Lyapunov technique, by constructing appropriate Lyapunov func ationals and finally by using a method based in Gronwall-like inequalities. Then the global asymptotic behaviour of solutions can be analyzed by using the theory of attractors. As the delay terms are allowed to be very general, the statement of the problem becomes nonautonomous in general. For this reason, the theory of nonautonomous pullback attractors appears to be appropriate.

On the existence of solutions for a nonconvex hyperbolic differential inclusion of third order

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We consider a Darboux problem associated to the following third order hyperbolic differential inclusion

$$u_{xyz}(x,y,z) \in F(x,y,z,u(x,y,z)), \qquad (x,y,z) \in \Pi,$$

where $\Pi := [0,T_1] \times [0,T_2] \times [0,T_3]$ and $F\colon \Pi \times \mathbb{R}^n \to \mathcal{P}(\mathbb{R}^n)$ is a set-valued map.

Our aim is twofold. On one hand, we show that Filippov's ideas can be suitably adapted in order to obtain the existence of a solution of problem (1)–(3). We recall that for a first order differential inclusion defined by a Lipschitzian set-valued map with nonconvex values Filippov's theorem consists in proving the existence of a solution starting from a given "almost" solution. Moreover, the result provides an estimate between the starting "quasi" solution and the solution of the differential inclusion. On the other hand, we prove the existence of solutions continuously depending on a parameter for problem (1)–(3). The key tool in the proof of this theorem is a result of Bressan, Colombo and Fryszkowski concerning the existence of continuous selections of lower semicontinuous multifunctions with decomposable values. This result allows to obtain a continuous selection of the solution set of the problem considered.

Traveling wave fronts in a coupled delayed reaction–diffusion and difference system

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The formation and development of blood cells (red blood cells, white cells and platelets) is a very complex process, called hematopoiesis. This process involves a small population of cells called hematopoietic stem cells (HSCs). We propose a mathematical model describing the dynamics of HSC population, taking into account their spatial distribution and diffusion. The resulting model is an age-structured reaction–diffusion system. The method of characteristics can be used to reduce this model to an unstructured time-delayed reaction–diffusion equation coupled with a difference equation. We investigated mathematical studies of the model and showed the existence of travelling wave front solutions connecting the zero equilibrium with the positive uniform steady state. We used the classical monotone iteration technique coupled with the sub- and super-solutions method. A numerical simulation was carried out to show the propagation of the solution in a travelling wave front.

Periodic trajectories of the linear swinging model

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This is a joint work with Prof. László Hatvani. The equation

$$x'' + a^2(t)x = 0,$$

$$a(t) := \begin{cases} \sqrt{\dfrac{g}{l-\varepsilon}} & \text{if } 2kT \le t < (2k+1)T, \\[2ex] \sqrt{\dfrac{g}{l+\varepsilon}} & \text{if } (2k+1)T \le t < (2k+2)T, \quad (k = 0, 1, \dots) \end{cases}$$

is considered, where g and l denote the constant of gravity and the length of the pendulum, respectively; $\varepsilon > 0$ is a parameter measuring the intensity of swinging. Concepts of solutions going away from the origin and approaching to the origin are introduced. Necessary and sufficient conditions are given in terms of T and ε for the existence of solutions of these types, which yield conditions for the existence of $2T$-periodic and $4T$-periodic solutions as special cases. The domain of instability, i.e. the Arnold tongues of parametric resonance are deduced from these results.

Numerical stability for nonlinear evolution equations

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The talk is about discretisation methods for nonlinear operator equations written as abstract nonlinear evolution equations. Brezis and Pazy showed that the solution of such problems is given by nonlinear semigroups whose theory was founded by Crandall and Liggett. By using the approximation theorem of Brezis and Pazy, we show the N-stability of the abstract nonlinear discrete problem for the implicit Euler method. Motivated by the rational approximation methods for linear semigroups, we propose a more general time discretisation method and prove its N-stability as well. The talk is based on the joint work with I. Faragó and I. Fekete.

Exponential stability of difference equations with distributed delay

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The aim of this presentation is to propose new conditions for stability for system of difference equations with distributed delay. In particular, we focus on the special case of system with single delay and we address the exponential stability of this system, by using the Lyapunov–Krasovskii approach. So, we obtain sufficient condition of the form of linear matrix inequality (LMI). Our approach is illustrated by some numerical examples.

Global stability for SIR and SIRS models via Dulac functions

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We prove the global asymptotic stability of the disease-free and the endemic equilibrium for general SIR and SIRS models with nonlinear incidence. Instead of the popular Volterra-type Lyapunov functions, we use the method of Dulac functions, which allows us to extend the previous global stability results to a wider class of SIR and SIRS systems, including nonlinear (density-dependent) removal terms as well. We show that this method is useful in cases that cannot be covered by Lyapunov functions, such as bistable situations. We completely describe the global attractor even in the scenario of a backward bifurcation, when multiple endemic equilibria coexist.

Joint work with Gergely Röst.

Approximation of distributed parameter systems by delay systems: a control perspective

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The present work addresses continuous-time approximation of distributed parameter systems governed by linear one-dimensional partial differential equations. While approximation is usually realized by lumped systems, that is finite dimensional systems, we propose to approximate the plant by a time-delay system. Within the graph topology, we prove that, if the plant admits a coprime factorization in the algebra of BIBO-stable systems, any linear distributed parameter plant can be approximated by a time-delay system, governed by coupled differential-difference equations. Considerations on stabilization, performances and state-space realization are carried out. A numerical method for constructive approximation is also proposed and illustrated.

Uniform exponential stability conditions for systems of linear delay differential equations

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Uniform exponential stability of linear systems with time varying coefficients

$$\dot{x}_i(t) = -\sum_{j=1}^{m}\sum_{k=1}^{r_{ij}} a_{ij}^k(t)x_j(h_{ij}^k(t)), \qquad i = 1, \dots, m,$$

is studied, where $t \geq 0$, m and r_{ij}, $i, j = 1, \dots, m$ are natural numbers, $a_{ij}^k\colon [0,\infty) \to \mathbb{R}$ and $h_{ij}^k\colon [0,\infty) \to \mathbb{R}$ are measurable functions. New results are derived with the proofs based on Bohl–Perron theorem. Several useful and easily verifiable corollaries are deduced and examples are provided to demonstrate the advantage of the stability results over known results.

Delay equations revisited

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If we define a DE (delay equation) as "a rule to extend a function of time towards the future on the basis of the (assumed to be) known past", both DDE (delay differential equations) and RE (renewal equations) are covered. For a given DE, one defines a semigroup of operators by shifting along the extended function. The question "how does the semigroup depend on the rule?" arises in the context of stability and bifurcation problems. The answer employs the variation-of-constants formula.

A first key idea is to describe the rule for extension by a cumulative output map [1] with finite dimensional range and to construct the extension, in the linear case, by solving a linear finite dimensional convolution equation. Once this is done, all that remains is to explicate the integral in the variation-of-constants formula.

Already in 1953 Feller [2] emphasized that one might use the Lebesgue integral for scalar valued functions of time obtained by pairing a linear semigroup acting on an element of the primal space with an element of the dual space and that there is no need to require strong continuity. More recently this point of view was elaborated by Kunze [3] who defined a Pettis type integral in the framework of a norming dual pair of spaces.

The aim of the lecture, which is based on joint work with S. M. Verduyn Lunel, is to derive in this manner a powerful version of the variation-of-constants formula for delay equations.

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Stability of delay differential systems

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In this talk we propose new explicit tests of exponential stability of delay systems. Our approach is based on nonoscillation of solutions and positivity of the Cauchy functions of corresponding scalar delay differential equations. Our results can be applied to exponential stabilization of systems with state and input time-varying large delays by feedback control with delays that can be greater than the state ones. In our approach unstable systems with positive delayed terms, implying oscillation and instability of diagonal equations and, as a result, instability of corresponding systems, are "compensated" by negative terms with close delays. Such a stabilization is always possible in the case where the control appears in each differential equation of the system. Results on stabilization on the basis of only one controlled component are also proposed. Several applications are demonstrated.

Asymptotic problems for fourth order differential equations

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Consider the fourth order differential equation with a deviating argument

$$x^{(4)}(t) + q(t)x^{(2)}(t) + r(t)|x(\varphi(t))|^{\lambda}\operatorname{sgn} x(\varphi(t)) = 0, \qquad (t \geq 0) \tag{1}$$

where $\lambda > 0$, $q, r, \varphi \in C[0, \infty)$, $\lim_{t\to\infty} \varphi(t) = \infty$ in case that the operator $x^{(4)}(t) + q(t)x''(t)$ is oscillatory. We discuss two asymptotic problems: the existence of asymptotically linear solutions and the oscillation of all solutions of (1).

This is a joint research with M. Bartušek (Masaryk University Brno), M. Cecchi and M. Marini (University of Florence).

Oscillation constants for a class of second-order ordinary differential equations

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We consider the second order differential equation

$$(t^{\alpha-1}\Phi(x'))' + t^{\alpha-1-p}f(x) = 0, \qquad \Phi(x) = |x|^{p-2}x,\ p > 1,\ \alpha \in \mathbb{R}, \tag{1}$$

with f satisfying $xf(x) > 0$, $x \neq 0$. When $f(x) = \lambda\Phi(x)$, $\lambda \in \mathbb{R}$, then (1) reduces to the classical half-linear Euler differential equation, whose oscillatory behavior is well understood.

We analyze the difference between the cases $\alpha < p$, $\alpha > p$ and $\alpha = p$. In each case we give a condition on the function f which guarantees that solutions of equation (1) are (non)oscillatory. The principal method used in the investigation is the Riccati technique and its modifications.

The presented results were achieved jointly with Naoto Yamaoka (Osaka, Japan).

Loss of immunity on epidemiological models with infection age

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Due to various environmental changes such as diversity of transport network and urbanization arising from a rapid increase in population, disease transmission has been considered as a serious threat to global population despite medical development. In view of mathematical biology, the epidemiological models play a crucial role to predict and qualitatively understand the eventual disease prevalence. In the literature, epidemic models with infection age (a time since infection has started) are widely formulated by means of renewal equations to investigate dynamical behavior of disease transmission more qualitatively. In this talk, we investigate asymptotic stability of equilibria of the models with the infection age incorporating the loss of immunity of recovered individuals. We establish the recent results and offer a several open problems for the stability of an endemic equilibrium when the basic reproduction number R_0 is greater than 1.

Functional Continuous Runge–Kutta–Nyström methods

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Numerical methods for solving retarded functional differential equations (RFDEs) of the second order with right-hand side independent of the function derivative are considered.

Second order differential equations can be rewritten as a first order system and standard Runge–Kutta methods can be applied to them. However, in case of ordinary differential equations, if their right-hand side does not depend on the first derivative, much more efficient methods named after E. Nyström can be constructed.

Special Runge–Kutta methods for direct application to RFDEs were first developed in the 1970's by Tavernini. But it is only in recent years that these methods have been expanded to a general class of methods called "Functional Continuous Runge–Kutta methods" for RFDEs. Functional Continuous Runge–Kutta methods are reviewed in Acta Numerica 2009, where order conditions and examples of methods are presented.

In this talk methods analogous to Runge–Kutta–Nyström methods are developed for the corresponding RFDE problems. The order conditions are formulated, and example methods are constructed. Application of the constructed methods to test problems confirms their declared orders of convergence.

Numerical modeling of epidemic propagation

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Most of the models of epidemic propagations do not take into the account the spatial distribution of the individuals. They give only the temporal change of the number of the infected, susceptible and recovered patients. In this paper we give some spatial discrete one-step iteration models for disease propagation and give conditions that guarantee some basic qualitative properties of the original process to the discrete models. Because the discrete models can be considered as the finite difference approximations of continuous models of disease propagation given in the form of systems of integro-differential equations, we can deduce conditions for the mesh size and the time step. Some of the results are demonstrated on numerical tests. This is joint work with R. Horváth.

Periodic solutions and bifurcation in generalized ODEs and applications

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By means of the coincidence degree theory, we show a result on the existence of a periodic solution and on the existence of a bifurcation point for periodic solutions of generalized ODEs. We translate our results to classic ODEs where the functions are merely Kurzweil–Henstock integrable coping, therefore, with many oscillations and jumps. These results are part of a joint work with J. Mawhin and M. C. Mesquita.

Schoenflies spheres in Sturm attractors

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Let v be a hyperbolic equilibrium of a smooth finite-dimensional gradient or gradient-like dynamical system. Assume that the unstable manifold W of v is bounded, with topological boundary $\Sigma = \partial W := (\text{clos } W) \setminus W$. Then Σ need not be homeomorphic to a sphere, or to any compact manifold.

In contrast, consider the scalar PDE

$$u_t = u_{xx} + f(x, u, u_x)$$

on a bounded interval. Then the decomposition of the global PDE attractor by unstable manifolds is a finite-dimensional regular CW-complex. The boundary Σ of each unstable manifold is homeomorphic to a sphere. In particular this excludes complications like lens spaces and Reidemeister torsion, or Alexander horned spheres.

Our results are based on Sturm nodal properties. Therefore analogous results may be expected for global attractors of delay equations with monotone feedback. Alas, the related PDE case of periodic boundary conditions is still open.

All results are joint work with Carlos Rocha (IST Lisbon). See also

`http://dynamics.mi.fu-berlin.de/`

Approximate solutions for delay differential equations

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Delay differential equations can be formally solved using the complex Lambert function, defined by:

$$W(h) \cdot e^{W(h)} = h.$$

This function has an infinite number of branches $W_p(h)$, almost all of which are complex. The solution is complete in terms of giving an expression for all exponential terms, and a formal procedure for finding their coefficients. However, the infinite expansion must always be truncated at some point. Our goal is to estimate the finite number of branches which make a significant contribution to the complete solution. In this work we study a scalar DDE, finding a systematic approximation scheme that indicates which branches should be included order to get an approximation satisfying certain criteria. The approximation scheme is then used for several different cases, showing to what extent the approximation is accurate.

Oscillatory behavior of a third-order neutral dynamic equation with distributed delays

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The authors present some new oscillation criteria for the third-order neutral dynamic equation with distributed delays

$$\left[r(t) \left(\left[x(t) + \int_a^b p(t,\eta) x\left[\tau(t,\eta)\right] \Delta\eta \right]^{\Delta\Delta} \right)^{\alpha} \right]^{\Delta} + \int_c^d q(t,\xi) f\left(x\left[\phi(t,\xi)\right]\right) \Delta\xi = 0$$

on a time scale $\mathbb{T}$, where α is the quotient of odd positive integers. Using a Riccati type transformation and a comparison technique, they establish some new sufficient conditions to ensure that a solution x of this equation either oscillates or satisfies $\lim_{t\to\infty} x(t) = 0$.

This is joint work with Said R. Grace of the Department of Engineering Mathematics at Cairo University in Orman, Giza, Egypt and Ercan Tunç of the Department of Mathematics at Gaziosmanpasa University in Tokat, Turkey.

On a logistic equation with delayed positive feedback

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In this talk we consider the qualitative properties of a logistic equation with two discrete delays – positive and negative delayed feedback. In the parameter plane we characterize the boundedness and global stability of the equilibrium. We will show that there exists an exponential solution for the nonlinear problem which is unbounded. We will discuss how positive and negative delayed feedback influence the dynamics. In a special case we completely characterize the stability and the boundedness. It will be shown that there exists a parameter set such that the solution is locally stable but not globally stable due to the existence of blow-up solutions. For a general case we will consider the existence of positive real roots from which we can deduce a part of the instability region for the two delay problem. This is a joint work with Yukihiko Nakata (University of Tokyo, Japan) and Gergely Röst (University of Szeged, Hungary).

Chemostat in varying environments

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Chemostat refers to a laboratory device used for growing microorganisms in a cultured environment, and has been regarded as an idealization of nature to study competition modeling in mathematical biology. The simple form of chemostat model assumes a stationary environment such that the availability of nutrient and its supply rate are both fixed. In addition the tendency of microorganisms to adhere to surfaces is neglected by assuming the flow rate is fast enough. However, these assumptions largely limit the applicability of chemostat models to realistic competition systems. In this work we relax these assumptions and study chemostat models in non-stationary environments with time dependent/random nutrient supplying rate or time dependent/random input nutrient concentration, with or without wall growth. This leads to nonautonomous/random dynamical systems and requires the concept of nonatutonomous/random attractors developed in the theory of nonautonomous/random dynamical systems. Our results include existence of uniformly bounded non-negative solutions, existence of attractors and geometric details of attractors for different value of parameters.

Nonlinear variation of constants formula for differential equations with state-dependent delays

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In this talk we consider a class of differential equations with state-dependent delays. We first discuss differentiability of the solution with respect to the initial function and the initial time assuming that the state-dependent time lag function is piecewise monotone. Based on these results, we prove a nonlinear variation of constants formula for differential equations with state-dependent delays. As an application, we discuss asymptotic properties of a perturbed nonlinear scalar differential equation with state-dependent delays.

Lyapunov–Razumikhin techniques for state-dependent DDEs

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We present theorems for the Lyapunov and asymptotic stability of the steady state solutions to general state-dependent delay differential equations (DDEs) using Lyapunov–Razumikhin methods. The Lyapunov stability result applies to nonautonomous DDEs with multiple discrete state-dependent delays of the form

$$\begin{cases} \dot{u}(t) = f(t, u(t), u(t-\tau_1(t,u(t))), \ldots, u(t-\tau_N(t,u(t)))), & t \geqslant t_0, \\ u(t) = \varphi(t), & t \leqslant t_0, \end{cases}$$

and is proved by a contradiction argument which is adapted from a previous result of Barnea for retarded functional differential equations (RFDEs). Our asymptotic stability result applies to autonomous DDEs with multiple state-dependent discrete delays. Its proof is entirely new, and is based on a contradiction argument together with the Arzelà–Ascoli theorem. This alleviates the need for an auxiliary function to ensure the asymptotic contraction, which is a feature of the other Lyapunov–Razumikhin asymptotic stability results of which we are aware. We apply our results to the state-dependent model equation

$$\begin{aligned} \dot{u}(t) &= \mu u(t) + \sigma u(t-a-cu(t)), & t \leqslant 0, \\ u(t) &= \varphi(t), & t \leqslant 0, \end{aligned}$$

which includes Hayes equation as a special case (when $c = 0$), to directly establish asymptotic stability in parts of the stability domain along with lower bounds for the basin of attraction. We also generalise our techniques to derive a condition for global asymptotic

stability of the zero solution to the model problem, and also to find bounds on the periodic solutions when the steady-state solution is unstable.

Joint work with Felicia Magpantay (U Michigan/U Manitoba).

Travelling waves for fully discretized bistable reaction–diffusion problems

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We study various temporal and spatial discretization methods for bistable reaction–diffusion problems. The main focus is on the functional differential operators that arise after linearizing around travelling waves in the spatially discrete problem and studying how the subsequent discretization of time affects the spectral properties of these operators. This represents a highly singular perturbation that we attempt to understand via a weak-limit method based on the pioneering work of Bates, Chen and Chmaj (2003). Once this perturbation is understood, one can study the existence and (non)-uniqueness of waves in the fully discretized reaction–diffusion system.

Revisiting the Kermack and McKendrick endemic model

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In a series of papers published during 1930s, although they have been paid very little attention in contrast with the famous outbreak model in 1927, Kermack and McKendrick have proposed infection-age structured endemic models, which take into account the demography of host population, the waning immunity, variable susceptibility and reinfection of recovered individuals [5, 6]. The idea of reinfection becomes more and more important to understand emerging and reemerging infectious diseases, since it makes the control of infectious diseases difficult, and the waning immunity (by natural decay or by relative decay due to genetic changes of infectious agents) is widely observed. According to Gomes, et al. [2], we can introduce the reinfection threshold of R_0 at which qualitative change in the epidemiological implication occurs for the prevalence and controllability. The aim of my talk is to uncover some unknown aspects of the reinfection phenomena and to show wide applicability of the age-structured endemic model.

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On the competition of species for territory with overcolonization

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We consider ODE-meanfield and cellular automaton models of some two-species metapopulations. The competing species can colonize not only empty patches but also overcolonize those which are occupied by the other. First, we investigate the dynamics of the ODE models with different ways of overcolonization (no interaction, hierarchical and general). We give conditions that either guarantee or exclude stability of the coexistence. Then, we show experimental results (using *Wolfram Mathematica*) for the analogous stochastic cellular automaton models. We investigate the dependence of the spatio-temporal development of the model on the colonization and extinction strategies as well as initial configurations.

Finite-time nonautonomous bifurcation in impulsive systems

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The purpose of this study is to investigate nonautonomous bifurcation in impulsive differential equations. The impulsive finite-time analogues of transcritical and pitchfork bifurcation are provided.

Periodic solutions of a delayed Van der Pol oscillator

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We report on existence results of periodic solutions to a delayed Van der Pol oscillator. We also present the main ingredients of a rigorous computational method that we used for establishing our results. Join work with Jean-Philippe Lessard, Laval University, Quebec, Canada.

Chaos by neural networks: the quasi-periodic route

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In this talk, we take into account neural networks that possess chaos with infinitely many unstable quasi-periodic motions in the basis. Retarded shunting inhibitory cellular neural networks and Hopfield neural networks are utilized for the chaos generation and extension processes. The presence of chaotic motions as well as the stabilization of quasi-periodic solutions are demonstrated though examples. The results may be applicable in health care areas such as medical image analysis, sleep apnea detection, cancer treatment, speech/auditory signal recognition and processing, and in many prediction tasks such as protein secondary structure and protein solvent accessibility.

Higher order Grünwald approximations of fractional derivatives and fractional powers of operators

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We give stability and consistency results for higher order Grünwald-type formulae used in the approximation of solutions to fractional-in-space partial differential equations. We use a Carlson-type inequality for periodic Fourier multipliers to gain regularity and stability results. We then generalise the theory to the case where the first derivative operator is replaced by the generator of a bounded group on an arbitrary Banach space. This is a joint work with Boris Baeumer and Harish Sankaranarayanan.

Lyapunov functions for a spatially diffusive SIR epidemic model

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To construct Lyapunov functions for partial differential equations as heterogeneous epidemic models is often a nontrivial task. The discretization of such models to ordinary differential equations is thought to be an effective way to obtain a suggestion of the form of a Lyapunov function since the theory of Lyapunov functions for multi-dimensional ordinary differential equations has recently been enriched by many authors. In this talk, I shall introduce a construction method of Lyapunov functions based on the discretization for a spatially diffusive SIR epidemic model.

State-dependent delays which yield complicated motion, Part II.

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This talk is connected to the plenary talk of H.-O. Walther (Part I), which described the construction of an equation with state dependent delay such that homoclinic behavior with a ‘minimal intersection property’ takes place.

Part II explains the geometry of a return map for that equation, and then employs a topological framework to obtain symbolic dynamics (in particular the Leray–Schauder fixed point index). The method is inspired by the covering relations as used by Polish colleagues, but different in that it separates the simplifying homotopy from the index computation. It applies also to a classical 4-dimensional example by L. P. Shilnikov; in both examples, it exhibits the essential topological properties.

On the oscillation of third order differential equations with delay

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Our aim is to study the oscillation and asymptotic properties of the third order nonlinear equations with delay of the form

$$\left(\frac{1}{p(t)}\left(\frac{1}{r(t)}x'\right)'\right)' + q(t)\,|x(g(t))|^{1/\lambda}\operatorname{sgn} x(g(t)) = 0 \tag{N, g}$$

and the adjoint equation

$$\left(\frac{1}{r(t)}\left(\frac{1}{p(t)}z'\right)'\right)' - q(t)\,|z(g(t))|^{\lambda}\operatorname{sgn} z(g(t)) = 0, \tag{N$^{\mathcal{A}}$, g}$$

where $0 \le \lambda \le 1$.

We will find conditions that every solution of equation (N$^{\mathcal{A}}$, g) is either oscillatory or satisfies $\lim_{t\to\infty}|z^{[i]}(t)| = \infty$, $i = 0, 1, 2$. Moreover, we will show some applications on equations with the symmetrical operator (i.e. $p(t) = r(t)$). In particular, we will give condition for equation

$$z''' - q(t)z^{\lambda}(g(t)) = 0$$

to have such property and we will improve conditions from [4] for the equation

$$z''' - \frac{\mu}{t^3}z(kt) = 0, \qquad (t \ge 1).$$

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Tensor products, positive operators, and delay-differential equations

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We study linear nonautonomous delay-differential equations, such as

$$\dot{x}(t) = -a(t)x(t) - b(t)x(t-1). \tag{$*$}$$

Such equations can occur as the linearization of a nonlinear delay equation

$$\dot{x}(t) = -f(x(t), x(t-1))$$

around a given solution (often around a periodic solution), and they are crucial in understanding the stability of such solutions. We develop an associated linear theory to equation ($*$) by taking the m-fold wedge product (in the infinite-dimensional sense of tensor products) of the dynamical system generated by ($*$). Remarkably, in the case of a "signed feedback," namely where $(-1)^m b(t) > 0$ for some integer m, the associated linear system is given by an operator which is positive with respect to a certain cone in a Banach space. This leads to detailed information about stability properties of ($*$), and in particular, information about its characteristic multipliers.

Periodic averaging for q-difference equations

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This is a joint work with Prof. Martin Bohner. The theory of averaging plays an important role for applications, since it can be used to study perturbation theory, control theory, stability of solutions, bifurcation, among others. In this work, we prove a periodic averaging principle for q-difference equations and present some examples to illustrate our result.

On some properties of lower triangular banded Toeplitz matrices corresponding to linear multistep methods

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We give a new proof of the stability of the explicit Euler method using M-matrix theory. Based on this idea we investigate linear multistep methods which result in lower triangular banded Toeplitz matrices. We investigate two questions: the first is to give an upper bound for the infinity norm of the inverse independently of the size of the matrix, the second is to decide under which conditions is the inverse nonnegative. These matrices are generally not Z-matrices, thus M-matrix theory is not applicable directly.

Dynamics of complex biological systems determined/controlled by minimal subsets of molecules in regulatory networks

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Modern biology provides many networks describing regulations between many species of molecules. It is widely believed that the dynamics of molecular activities based on such regulatory networks are the origin of biological functions. In this study we develop a new theory to provide an important aspect of dynamics from information of regulatory linkages alone. We show that the "feedback vertex set" (FVS) of a regulatory network is a set of "determining nodes" of the dynamics. It assures that i) any long-term dynamical behavior of the whole system, such as steady states, periodic oscillations or quasi-periodic oscillations, can be identified by measurements of a subset of molecules in the network, and that ii) the subset is determined from the regulatory linkage alone. For example, dynamical attractors possibly generated by a signal transduction network with 113 molecules can be identified by measurement of the activity of only 5 molecules, if the information on the network structure is correct. We also demonstrate that controlling the dynamics of the FVS is sufficient to switch the dynamics of the whole system from one attractor to others, distinct from the original.

Boundedness of solutions of nonlinear delay differential equations

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In our work we obtain sufficient conditions for the persistence, the permanence and the boundedness of the positive solutions of a large class of nonlinear differential equations with delays which can be considered as a population model with delayed birth term and instantaneous death term. Our results improve earlier conditions proved in the literature.

We have also given explicit conditions for the boundedness of the positive solutions of special cases of our general class of nonlinear delay differential equations, e.g., the equations

$$\dot{x}(t) = \frac{\alpha(t)x(t-\sigma(t))}{1+\gamma(t)x(t-\sigma(t))} - \beta(t)x^2(t), \qquad t \geq 0,$$
$$\dot{x}(t) = \alpha(t)f(x(t-\sigma(t))) - \beta(t)h(x(t)), \qquad t \geq 0,$$

and

$$\dot{x}(t) = \sum_{k=1}^{n} \alpha_k(t)x^p(t-\tau_k(t)) - \beta(t)x^q(t) \qquad t \geq 0.$$

Joint work with István Győri and Ferenc Hartung.

Stability analysis of multi-group SIS epidemic models and related models

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This talk deals with recent techniques from Lyapunov functional and monotone iterative approach to obtain complete global dynamics of some class of multi-group SIS epidemic models by a threshold parameter (the basic reproduction number). Applying these to the stability analysis of related models, we establish sufficient conditions of the global stability which improve the known results in literature.

Approximate master equations for dynamical processes on graphs

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Epidemic processes running on large networks attracted considerable interest in the last decade. Assuming a simple dynamics at the node level such as the SIS-model, when nodes can be susceptible or infected, leads to a stochastic process where the structure of the network will have an impact on how the infection spreads over the network. The mathematical model describing the process is a continuous time Markov chain with extremely large state space (2^N, where N is the number of nodes in the network), leading to the master equations that form a system of linear ordinary differential equations (consisting of 2^N equations). Solving these even numerically is impossible for the typical values of N simply due to the large number of equations. The challenge is to reduce the size of the state space from 2^N to $N + 1$, thus the new state space is $\{0, 1, \ldots, N\}$, denoting the number of infected nodes in the network. We approximate analytically the transmission rates and the reduced system gives good agreement with the exact model. We show that this approach is feasible for graphs with arbitrary degree distributions built according to the configuration model.

Malaria dynamics in seasonal environment with long incubation period in hosts

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The incubation period of malaria can vary depending on the species of parasite or the geographic regions. In particular, in endemic areas of temperate climate (for example in Korea), the incubation period of *Plasmodium vivax* shows bimodal distribution of short and long term incubation periods. Assuming fixed length for the long term incubation period (DDE) gives a distribution that is much closer to the empirical distribution in the most common probability metrics, than the exponentially distributed long term incubation period (ODE). In the first part of the talk, we compare two transmission models for *P. vivax* malaria, where we model the long term incubation period using ordinary differential equations or delay differential equations. We identify the basic reproduction number R_0 and show that it is a threshold parameter for the global dynamics of the model. We show that, while the qualitative behavior of the two models is similar, the ODE model overestimates the basic reproduction number and also the level of endemicity, compared to the DDE model. By calculating R_0, we can see that long incubation time is not beneficial to the parasite in a constant environment, thus its presence is connected to the seasonal mosquito

activity in Korea. In contrast to the autonomous case, when we incorporate seasonality into our model equations, the interplay of the time delay and the periodicity results that in some situations the DDE model predicts higher prevalence of malaria. The periodic DDE model is also superior to periodic ODE in capturing the qualitative properties of the observed Korean malaria time series, while its mathematical analysis is rather challenging. In the second part of the presentation, we consider the evolution of the incubation time. Prolonged incubation time of *P. vivax* malaria in temperate region is considered to be an adaptation strategy to the seasonal environment. We present evolutionary models of the pathogen in a seasonal environment. Using theories of adaptive dynamics, we explore the direction of the evolution depending on mosquito season length.

Mathematical modeling and simulation of HIV infection in a lymph node network

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The human immunodeficiency virus (HIV) is a fast replicating virus that finally induces immunodeficiency. The main target cell for HIV is CD4 positive cells which include helper T cells and macrophages. Lymph nodes are recognized as one of the primary sites of HIV infection in which CD4 positive T cells are staying. Despite the recent progress in real-time monitoring technology, HIV infection dynamics in a lymph node network are largely unknown. In this talk, we present our recent progress on mathematical modeling and simulation of HIV infection in a lymph node network. HIV infection in a lymph node network in our formulation is described as compartmentalized ordinary differential equations: infection takes places in each lymph node, and then spreads via migration of infected cells or HIV particles (virions). The basic reproduction number defined for our main model is calculated to quantitatively characterize the spread of HIV infection. We then perform numerical simulations to investigate the effect of combinational multi-drug therapy. One of important findings from our theoretical investigation is the presence of the limit of drug treatment. This might partly explain HIV persistence despite the existence of sufficient drug supply.

On a transcendental equation from an endemic model

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When one analyzes (in)stability of a positive equilibrium, where population of interest persists, often a complicated characteristic equation arises due to both positive and negative feedback to the system. The characteristic equation from delay equation is given by a transcendental equation. We would like to discuss distribution of zeroes of a class of a transcendental equation:

$$1 = \int_0^1 k(a) e^{-\lambda a} da,$$

where k can be decomposed into positive and negative parts. Our aim is to characterize the kernel k in terms of distribution of zeroes, from which we can conclude (in)stability for a class of structured population models. Stability boundaries in a two-parameter plane will be presented. Speculations for a difference equation in continuous time would be given.

Asymptotic stability of constant solutions in delay differential equations with a constant delay and transcendental equations with complex coefficients

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In this talk, we investigate the asymptotic stability of a constant solution for a delay differential equation (DDE) $x'(t) = f(x(t), x(t-\tau))$, where $f\colon \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is a smooth function, and τ is a positive constant. This is an equation with a single constant delay.

In general, the asymptotic stability of a constant solution of DDEs is determined by the location of the roots of the characteristic equation of the linearized equation. For the above DDE, the characteristic equation becomes $\det(\lambda I + A - \mathrm{e}^{-\lambda\tau} B) = 0$, where A and B are $n \times n$ real matrices. If A and B are simultaneously triangularizable, then this equation is reduced to a transcendental equation $z + a - b\mathrm{e}^{-z} = 0$, where parameters a and b are generally complex.

We see that by using the "graph-like" expression of the Lambert W function in some coordinate system of the complex plane $\mathbb{C}$, a necessary and sufficient condition on a and b for which all the roots of the above transcendental equation have negative real parts can be obtained. Here the Lambert W function is the multi-valued inverse of a complex function $z \mapsto z\mathrm{e}^z$, and the set of the roots is equal to $W(b\mathrm{e}^a) - a$. We also give an application of this result to the stabilization problem of unstable constant solutions by the delayed feedback control proposed by Pyragas.

Analyticity and nonanalyticity for solutions of "analytic" differential-delay equations

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We shall discuss conditions under which bounded solutions of "analytic" functional differential equations are necessarily everywhere real analytic; but we shall also describe examples for which bounded, infinitely differentiable solutions of superficially "analytic" differential-delay equations are real analytic on open intervals but also fail to be real analytic at many points, possibly even on Cantor-like sets. We shall mention some open conjectures: some very simple-looking "analytic" differential-delay equations which have bounded, infinitely differentiable solutions which are conjectured (no proofs!) to be nowhere real analytic. We shall also describe some connections between these results and the linear Krein–Rutman theorem.

Continuous separation for non-autonomous FDE with finite delay

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Classical techniques of topological dynamics are used to prove a flow extension result for linearly stable minimal sets in monotone and differentiable skew-product semiflows generated by non-autonomous FDE. We introduce a concept of continuous separation applicable in the context of FDE and PFDE with finite delay. These results are relevant in the study of uniform persistence and in the calculus of the upper Lyapunov exponent.

Weighted inequality and some qualitative characteristics of Sturm–Liouville type equation

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Let $I = (a,b) \subseteq R$. On the set of smooth and finitely supported functions on I we find necessary and sufficient conditions for the validity of the inequality:

$$\int_0^\infty w(t)|f(t)|^p dt \le C \int_a^b \left(\rho(t)|f'(t)|^p + r(t)|f(t)|^p\right) dt, \qquad f \in \mathring{W}^1_p(\rho, r; I). \tag{1}$$

where w, r and ρ are weight functions.

The obtained results we apply to establish oscillation properties of the equation in the form:

$$-(\rho(t)|y'(t)|^{p-2}y'(t))' + r(t)|y(t)|^{p-2}y(t) - w(t)|y(t)|^{p-2}y(t) = 0,$$

and when $p = 2$ to find the spectral characteristics of Sturm–Liouville equation, where $1 < p < \infty$, $\rho > 0$, $w \ge 0$ and $r \ge 0$ are continuous functions.

Inequality (1) is a partial case of the inequalities considered earlier in the papers []. However, the equivalence coefficients of the best constant in (1) are not pointed out in these works. Here we investigate inequality (1) by a method that allows us to find the equivalence coefficients more precisely.

The results are obtained jointly with A. Tiryaki and Kh. S. Ramazanova

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Complete second order differential operator with unbounded drift

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In spectral theory of differential operator, an important direction is to study the problems: when spectral set of differential operator is discrete set and its inverse belong to some Schatten classes? In the present report, we consider these problems for a class of operators

$$ly := -y'' + r(x)y' + q(x)y$$

acting in $L_2(R)$, where r and q are sufficiently smooth functions. We assume that $|r|$ has a faster growth at infinity than q, and q is not limited from below.

Well known that the class of operators l used to describe various problems of oscillation theory and stochastic processes.

Asymptotic inference for a stochastic differential equation with uniformly distributed time delay

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For the affine stochastic delay differential equation

$$\mathrm{d}X(t) = a \int_{-1}^{0} X(t+u)\,\mathrm{d}u\,\mathrm{d}t + \mathrm{d}W(t), \qquad t \geq 0,$$

local asymptotic properties of the likelihood function are studied. Local asymptotic normality is proved in case of $a \in \left(-\frac{\pi^2}{2}, 0\right)$, local asymptotic mixed normality is shown if $a \in (0, \infty)$, periodic local asymptotic mixed normality is valid if $a \in \left(-\infty, -\frac{\pi^2}{2}\right)$, and only local asymptotic quadraticity holds at the points $-\frac{\pi^2}{2}$ and 0. Applications to the asymptotic behaviour of the maximum likelihood estimator $\widehat{a}_T$ of a based on $(X(t))_{t\in[0,T]}$ are given as $T \to \infty$.

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Memory-dependent growth rates in sublinear Volterra differential equations

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We study the asymptotic growth of solutions of the functional integro-differential equation

$$x'(t) = \int_0^t \mu(ds) f(x(t-s)), \qquad t > 0;\ x(0) = \xi > 0, \tag{1}$$

where μ is a positive measure and f is a positive function obeying $\lim_{x\to\infty} f(x)/x = 0$. Using comparison methods we characterise the rate of growth of solutions in terms of related ordinary differential equations. When μ is a finite measure on $\mathbb{R}^+$ we obtain general results for asymptotically monotone nonlinearities. To deal with the infinite measure case we exploit the powerful theory of regular variation. In this framework, studying the equation as a Volterra convolution equation yields precise asymptotic results. We uncover explicit memory dependence in the growth rate of the solution and second order effects from both the nonlinearity and the memory. In particular, faster decay in the measure leads to faster growth in the solution.

This talk is based on joint work with Prof. John Appleby (DCU) and is supported by the Irish Research Council under the project GOIPG/2013/402.

System of delay difference equations with continuous time with lag function between two known functions

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The asymptotic behaviour of solutions of the system of difference equations with continuous time which lag function is between two known real functions is studied. The cases when the lag function is between two pantograph-type delay functions, between two power delay functions and between two constant delay functions are observed and illustrated by examples. The asymptotic estimates of solutions of the considered system are obtained.

Asymptotic results for a linear delay differential equation

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We will consider the linear delay differential equation

$$x'(t) = p(t)x(t-r),$$

where $r > 0$ and $p\colon [t_0, \infty) \to \mathbb{R}$ is a continuous function which tends to zero as $t \to \infty$. Recently, we have shown that every solution of the above equation satisfies the asymptotic relation

$$x(t) = \frac{1}{y(t)}\big(\, c + o(1)\,\big), \qquad t \to \infty, \tag{1}$$

where y is an eventually positive solution of the associated formal adjoint equation with bounded growth and c is a constant depending on x. In this talk we give a description of the special solution y of the formal adjoint equation which yields explicit asymptotic formulas for the solutions of the above delay differential equation. This is a joint work with István Győri (University of Pannonia, Hungary) and Gergely Röst (University of Szeged, Hungary).

A time-discontinuous Galerkin method for neural field models with transmission delays

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Neural field equations are models that describe the spatio-temporal evolution of (spatially) coarse grained variables such as synaptic or firing rate activity in populations of neurons. We consider p interacting populations of neurons, distributed over some bounded, connected, open region, whose state is described by their membrane potential V_i, $i = 1, \dots, p$. These potentials are assumed to evolve according to the system of integro-differential equations:

$$\frac{\partial V_i}{\partial t}(t,r) = -\alpha_i V_i(t,r) + \sum_{j=1}^{p} \int_\Omega J_{ij}(r,r')S_j(V_j(t-\tau_{ij}(r,r'),r'))d\,r'.$$

The numerical treatment of these systems is rare in the literature and has several restrictions on the space domain and the functions involved. We propose the use of a time-discontinuous Galerkin finite element method, which is well established to solve partial differential equations. We show that long time integration is possible without error accumulation, hence the method is particularly suitable when the long time behavior of the solution is of interest.

This is a joint work with J. J. W. van der Vegt (University of Twente).

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The role of the dichotomy spectrum in nonautonomous bifurcations

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The dichotomy spectrum (also known as dynamical or Sacker–Sell spectrum) is a crucial notion in the theory of nonautonomous dynamical systems. Besides information on stability and robustness properties, it is fundamental to establish a geometric theory including invariant manifolds, foliations, (topological) linearization and normal forms.

However, recent applications in nonautonomous bifurcation theory showed that a detailed insight into the fine structure of this spectral notion is necessary. On this basis, we explore a helpful connection between the dichotomy spectrum and operator theory. It relates the asymptotic behavior of linear nonautonomous equations to the (approximate) point, surjectivity and Fredholm spectra of weighted shifts. This link yields several dynamically meaningful subsets of the dichotomy spectrum, which

(a) allow to classify nonautonomous bifurcations on a linear basis already

(b) simplifies proofs for results on the long term dynamics of difference and differential equations with explicitly time-dependent right-hand side

(c) provides sufficient conditions for a continuous (rather than merely an upper-semicontinuous) behavior of the dichotomy spectrum under perturbation.

Discrete model of population dynamics of Easter Island and extinct populations

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We investigate the local stability character of continuous and discrete population models of Easter Island; in addition, we investigate the global behavior of solutions and different types of bifurcations. Furthermore, we compare the similarities and differences between the continuous and discrete systems.

Sturm type comparison theorems for the half-linear equations with damping term

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Joined with Tiryaki A. and Oinarov R.

Let $I = (a, b) \in \mathbf{R}$. Then we consider a class of half-linear equations with damping term of the form

$$\ell u := \Big(p_1(x)\varphi(u')\Big)' + q_0(x)\varphi(u') + p_0(x)\varphi(u) = 0, \tag{1}$$

$$Lv := \Big(P_1(x)\varphi(v')\Big)' + Q_0(x)\varphi(v') + P_0(x)\varphi(v) = 0, \tag{2}$$

where $\varphi(z) = |z|^{\alpha-1}z, \alpha > 0$.

If $\alpha = 1$ then we get that the linear equations with damping term $\ell u = 0$ and $Lv = 0$. The based on the Picone's identity and inequality have obtained the difference the qualitative characteristics of their solutions for equations $\ell u = 0$ and $Lv = 0$ (see [1]). When $q_0 = Q_0$, the Picone's identity and inequality have got for the equations (1) and (2) [2].

Here we get that the Picone type identity and inequality for equations (1) and (2) without $q_0 = Q_0$ which established the different properties solutions for equations (1) and (2). In particular

Theorem 1. *Assume that* $(\alpha+1)P_1(x) > |Q_0(x)|$. *If there exists a solution* v *of* $Lv = 0$ *such that* $v(x) \neq 0$ *on* (x_1, x_2), *then* $J(\eta) \geq 0$, *for all* $\eta \in U$, *except for a possible* $v(x_1) = v(x_2) = 0$ *and* $\eta = Kve^{-\int_{x_0}^{x} \frac{|Q_0(s)|}{(\alpha+1)P_1(s)} \frac{dv}{v}}$, $K > 0$, *where*

$$J(\eta) = \int_{x_1}^{x_2} \left\{ \left(\frac{P_1^{\alpha+1}(x)}{\left(P_1(x) - \frac{|Q_0(x)|}{\alpha+1}\right)^{\alpha}} + \frac{2\alpha}{\alpha+1}|Q_0(x)| \right) |\eta'|^{\alpha+1} - \left(P_0(x) - \frac{3}{\alpha+1}|Q_0(x)| \right) |\eta|^{\alpha+1} \right\} dx,$$

and U*—be the set of all real values functions.*

Corollary 2. *Assume that* $(\alpha+1)P_1(x) > |Q_0(x)|$. *If there exists an* $\eta \in U$ *such that* $J(\eta) \leq 0$ *then every solution* v *of* $Lv = 0$ *has a zero in* (x_1, x_2) *except possibly* $v(x_1) = v(x_2) = 0$ *and* $\eta = Kve^{-\int_{x_1}^{x} \frac{|Q_0(s)|}{(\alpha+1)P_1(s)} \frac{dv}{v}}$.

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On functional differential equations associated to controlled structures with propagation

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The method of integration along the characteristics has turned to be quite fruitful for qualitative analysis of physical and engineering systems described by large classes of partial differential equations of hyperbolic type in the plane (time and one space dimension) with real characteristics. In this talk there is presented an overview of such models under the aforementioned approach. We mention in this abstract the models describing transport phenomena (e.g. circulating fuel nuclear reactors and tubular reactors of the biotechnology) and propagation phenomena (e.g. electrical transmission lines such as waveguides or water, steam and gas pipes).

What makes the difference is the number of waves and/or, consequently, the number of distinct families of characteristics. In the first case (transport phenomena) there exists a single forward (progressive) wave due to the fact that there exists a single family of characteristics which are increasing. In the second case (propagation) there are to be met both forward (progressive) and backward (reflected) waves and two families of characteristics. In the nonlinear case the systems of conservation laws belong to both categories of systems.

Integration along the characteristics allows association of some systems of functional (differential) equations; a one-to-one (injective) correspondence between the solutions of the two mathematical objects (the boundary value problem for the partial differential equations and the functional equations) is established such that all properties obtained for one of them is projected back on the other. In this way continuous and discontinuous classical solutions can be analyzed from the point of view of the well-posedness in the sense of Hadamard (existence, uniqueness and data/parameter dependence), existence of some invariant sets and stability.

The various functional equations thus introduced are mathematical objects interesting for themselves such as the neutral functional differential equations which appear in lossless and distortionless wave propagation when differential equations are to be met in the boundary conditions.

Integral stability for dynamic systems on time scale

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We consider the dynamic system in a Banach space on unbounded above and below time scale:

$$\begin{cases} x^{\Delta} = A(t)x + f(t,x,y), \\ y^{\Delta} = B(t)y + g(t,x,y). \end{cases} \tag{1}$$

This system satisfies the conditions of integral separation with the separation constant ν; the integral contraction with the integral contraction constant λ, nonlinear terms are ε-Lipschitz, and the system has a trivial solution.

We prove the theorem of asymptotic phase. Using this result and the centre manifold theorem we reduce the investigation of integral stability of the trivial solution of (1) to investigation of integral stability of the trivial solution of the reduced dynamic system

$$x^{\Delta} = A(t)x + f(t,x,u(t,x)). \tag{2}$$

This work was partially supported by the grant Nr. 345/2012 of the Latvian Council of Science.

Local and asymptotic properties of solutions to parabolic partial differential equations with state-dependent delays

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We study a class of parabolic nonlinear evolution equations with discrete state-dependent delay. It is well-known that reaction terms with discrete state-dependent delay are more sensitive than the ones with distributed delays. It requires an extra care when choosing the phase space. We are interested in the well-posedness of the initial value problem in different spaces. Several possible choices of phase spaces are presented. We also study the existence of global and exponential attractors. The recently developed method of quasi-stability estimates allows us to prove that in some cases the attractors are of finite fractal dimension.

Some interesting features of periodic delay differential equations

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In this talk we discuss some interesting dynamical phenomena of time periodic delay differential equations. In the first part (which is joint work with Kyeongah Nah), we consider a scalar linear equation with periodic coefficients, and assuming a sign-keeping property, we give a sufficient and necessary condition for the stability of zero. We construct specific examples to show that an intuitive criterion for the stability can easily fail, and explore stability switches for various ratios of the delay and the period. Second, we show that any discrete dynamics can be realized by scalar periodic delay differential equations, and for an arbitrary map we explicitly construct the periodic equation such that its Poincaré-map generates a topologically equivalent dynamics to the n-dimensional dynamics generated by the iterates of the original map. Finally, we show how some real world phenomena, such as malaria dynamics in temperate regions and irregular cholera outbreaks can be explained by periodic delay differential equations.

Global dynamics of populations with competition among immature individuals

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We analyze a population model for two age-structured species allowing for inter- and intra-specific competition at immature life stages. The dynamics is governed by a system of delay differential equations (DDEs) recently introduced by Gourley and Liu. The analysis of this model presents serious difficulties because the right-hand sides of the DDEs are defined in terms of the Poincaré map of a system of ODEs that in general cannot be solved explicitly. Using the notion of strong attractor, we reduce the study of the attracting properties of the equilibria of the DDEs to the analysis of a related two-dimensional discrete system. Then, we combine some tools for monotone planar maps and planar competing Lotka–Volterra systems to describe the dynamics of the model with three different birth rate functions: linear, rational (of Beverton–Holt type), and exponential (of Ricker type). We give easily verifiable conditions for global extinction of one or the two species, and for global convergence of the positive solutions to a coexistence state. This is a joint work with E. Liz.

Topological decoupling and linearization of nonautonomous reaction–diffusion equations

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Joint work with Christian Pötzsche (Alpen-Adria-Universität Klagenfurt)

Topological linearization results typically require the semi-flow under consideration to be invertible. An exception occurs when the spectrum of the linearization fulfills appropriate assumptions met e.g. for scalar reaction–diffusion equations. In this talk, we extend earlier work by Lu [1] to nonautonomous evolution equations under corresponding assumptions on the Sacker–Sell spectrum of the linear part.

[1] K. Lu, A Hartman–Grobman theorem for scalar reaction–diffusion equations. *J. Differential Equations* **93**(1991), 364–394.

Sturm comparison theorems for some elliptic type equations via Picone type inequalities

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In this talk we present some recent developments for half-linear type elliptic equations. We obtain Picone-type inequalities for pair of elliptic type equations with damping and external terms to establish Sturmian comparison theorems. Some oscillation results are given as an application.

Joint work with Aydın Tiryaki and Emine Mısırlı.

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On the Fučík type problem with nonlocal integral boundary conditions

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We consider the problem

$$-x'' = \mu x^+ - \lambda x^-, \qquad t \in [0,1], \tag{1}$$

$$x(0) = \gamma_1 \int_0^1 x(s)ds, \tag{2}$$

$$x(1) = \gamma_2 \int_0^1 x(s)ds \tag{3}$$

where the boundary conditions are of the form (2), (3). As a motivation for our work we mention the paper [1], where similar problem is treated for the case $\mu = \lambda$. We wish also to generalize the results in paper [2], where $\gamma_1 = 0$, that is, where the integral condition is given only on one interval end point.

We suggest spectra for the case $\gamma_1 = \gamma_2$. Visualizations and discussions are proposed also.

This is a joint work with S. Pečiulytė from Vytautas Magnus University.

This work was partially supported by the ESF project
2013/0024/1DP/1.1.1.2.0/13/APIA/VIAA/045.

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[2] N. Sergejeva, On some problems with nonlocal integral condition, *Math. Model. Anal.* **15**(2010), No. 1, 113–126.

On a system of nonlinear partial functional differential equations

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We consider a system of a semilinear hyperbolic functional differential equation (where the lower order terms contain functional dependence on the unknown function) with initial and boundary conditions and a quasilinear elliptic functional differential equation (containing t as a parameter) with boundary conditions. Existence of solutions for $t \in (0, T)$ and for $t \in (0, \infty)$ will be shown and some qualitative properties of the solutions in $(0, \infty)$ will be formulated.

Modelling epidemic propagation on heterogeneous networks by simple systems of differential equations

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The well-known ODE model for SIS (susceptible–infected–susceptible) epidemic propagation was developed for regular random networks (where each node has the same degree). This model is formulated in terms of the number of infected nodes and the number of SI and SS edges, hence it is called the pairwise model. For heterogeneous networks (with a given degree distribution) the heterogeneous pairwise model was introduced, however, the size of this system is of order K^2, where K is the number of different degrees in the network. Thus for random graphs with long tail degree distribution (such as power-law random graphs) this system is too large for numerical investigation. An approximating system of size order K, called compact pairwise model, was developed to overcome this difficulty. In this talk an even simpler approximating model, containing only four differential equations, is presented. This system is based on a new closure relation incorporating the second and third moment of the degree distribution.

Product integration and linear differential equations in Banach algebras

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The concept of product integration was originally introduced by V. Volterra: Given a continuous matrix-valued function $A\colon [a,b] \to \mathbb{R}^{n\times n}$, he considered products of the form

$$(I + A(\xi_m)(t_m - t_{m-1}))(I + A(\xi_{m-1})(t_{m-1} - t_{m-2})) \cdots (I + A(\xi_1)(t_1 - t_0)), \qquad (1)$$

where $a = t_0 < t_1 < \cdots < t_m = b$ and $\xi_i \in [t_{i-1}, t_i]$, $i \in \{1, \ldots, m\}$. The product integral $\prod_a^b (I + A(t)\,\mathrm{d}t)$ is then defined as the limit of the product (1) when the lengths of all subintervals $[t_{i-1}, t_i]$ approach zero. The motivation for introducing this concept stems from the fact that the indefinite product integral $t \mapsto \prod_a^t (I + A(s)\,\mathrm{d}s)$, $t \in [a,b]$, corresponds to the fundamental matrix of a system of n homogeneous linear ordinary differential equations $x'(t) = A(t)x(t)$.

In this talk we focus on Kurzweil's modification of the original definition, which considerably extends the class of product integrable functions. Moreover, we consider functions A with values in an arbitrary unital Banach algebra. In the infinite-dimensional case, the Kurzweil product integral loses some of its pleasant properties. To overcome this difficulty, we introduce a new concept of the strong Kurzweil product integral, and describe its relation to linear differential equations in Banach algebras.

Reaction–diffusion equations on lattices and graphs

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In this talk we discuss the reaction diffusion equation

$$\partial_t u = k\partial_{xx} u + f(u)$$

on various time and space structures. Namely, we assume that time is either discrete or continuous and the space is an arbitrary lattice or a graph. We show how certain properties (e.g., maximum principles, existence of solutions) depend on the time scale and graph/lattice characteristics.

This is a joint work with Jonáš Volek.

Representation of solutions of second-order linear differential systems with constant delays

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In the talk a new representation of the solution to initial problem

$$x''(t) - 2Ax'(t-\tau) + (A^2 + B^2)x(t-2\tau) = \theta, \qquad t \in (0, \infty),$$
$$x(t) = \xi(t), \qquad t \in [-2\tau, 0],$$

where A, B are $n \times n$ constant commuting matrices, $x\colon [-2\tau, \infty] \to \mathbb{R}^n$, $\xi\colon [-2\tau, 0] \to \mathbb{R}^n$ and θ is n-dimensional zero vector, is given.

Constructing Poincaré maps for delay differential equations using rigorous interval arithmetic

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Rigorous construction of Poincaré maps is a key ingredient in many computer assisted proofs in dynamical systems, especially in the context of existence of fixed points, periodic orbits and chaotic dynamics.

In this talk I would like to present (briefly) the methodology used to construct rigorous integrator for delay differential equations with constant delay (DDEs) and how to use it to obtain estimates of the images of a Poincaré map defined on a local affine section in some finite dimensional representation of the phase space. I will discuss applicability of the proposed method in computer assisted proofs of the existence of periodic orbits in the given DDE. I will also highlight some problems that may be encountered when looking for a good candidate for the section and the guess on the estimates for the initial set.

Vibrating infinite string under a minimally smooth force and a general observation condition

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The observation problem for the vibrating string is to find the initial data for which some prescribed (or observed) partial state (e.g. the position or the speed of the string) are attained at two time instants $t_1,\ t_2$.

Existence of classical solution $u(x,t) \in C^2(\mathbb{R}^2)$ to the following problem is proved:

$$Lu := u_{tt}(x,t) - a^2 u_{xx} = f(x,t), \qquad (x,t) \in \mathbb{R}^2,\ a > 0, \tag{1}$$

under the observation conditions (the observed states) given at $t_1,\ t_2 \in \mathbb{R}$ with variable coefficients $A_1,\ B_1,\ A_2,\ B_2$

$$\begin{aligned} A_1(x)u|_{t=t_1} + B_1(x)u_t|_{t=t_1} &= g_1(x), \qquad x \in \mathbb{R}, \\ A_2(x)u|_{t=t_2} + B_2(x)u_t|_{t=t_2} &= g_2(x), \qquad x \in \mathbb{R}. \end{aligned} \tag{2}$$

Here the coefficients $A_i,\ B_i,\ i = 1,2$, and $g_1,\ g_2$ are given functions smooth enough, the forcing function $f \in C(\mathbb{R}^2)$ and the directional derivative $\partial f/\partial \nu$ exists for some direction ν transversal to the characteristics, and $\partial f/\partial \nu \in C(\mathbb{R}^2)$.

On a nonlocal resonant boundary value problem involving p-Laplacian

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We study the existence of solutions for the following system of nonlocal resonant boundary value problem

$$(\varphi(x'))' = f(t,x,x'), \qquad x'(0) = 0, \qquad x(1) = \int_0^1 x(s)dg(s),$$

where $f\colon [0,1] \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is continuous and $g\colon [0,1] \to \mathbb{R}^n$ is a function of bounded variation. Moreover, we assume that

$$\varphi(s) = \psi_p(s),$$

where $\psi_p(s) = |s|^{p-2}s$, for $s \neq 0$, $\psi_p(0) = 0$, $s \in \mathbb{R}^n$, $p > 1$, or

$$\varphi(s) = (\phi_{p_1}(s_1), \dots, \phi_{p_n}(s_n)),$$

$i = 1, \dots, n$, $s \in \mathbb{R}^n$, $p_i > 1$ and $\phi_{p_i}\colon \mathbb{R} \to \mathbb{R}$ is the one dimensional p_i-Laplacian.

The proofs of the main results are depend upon the coincidence degree theory.

Stability of intracellular delay, immune activation delay and nonlinear incidence on viral dynamics

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This presentation considers a class of viral infection models with two type discrete delays, one of which represents an intracellular latent period for the contacted target cells with viruses to begin producing virions, the other of which represents a time delay needed in cytotoxic T cells (CTLs) immune response before immune becomes effective after the invasion by a novel pathogen. By constructing Lyapunov functionals we investigate the global stability of the equilibria.

New configurations of periodic orbits for delay equations with positive feedback

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This talk considers scalar delay differential equations of the form

$$\dot{x}(t) = -\mu x(t) + f(x(t-1)),$$

where $\mu > 0$ and f is a nondecreasing C^1-function. We say that a periodic solution has large amplitude if it oscillates about at least two unstable equilibria. We investigate what type of large-amplitude periodic solutions may exist at the same time when the number of unstable equilibria is arbitrarily large.

If time permits, we characterize the geometrical properties of the unstable set of a large-amplitude periodic orbit which oscillates about two unstable equilibria.

The spectral theory for periodic delay equations revisited

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In this talk we present necessary and sufficient conditions for completeness of the span of eigenvectors and generalized eigenvectors for a large class of linear operators including Hilbert–Schmidt operators of order one.

The results are based on the notion of a characteristic matrix and precise resolvent estimates near infinity using the Phragmén–Lindelöf indicator function. Our result extends Keldysh type theorems to operators that are not close to selfadjoint. Several concrete applications, in particular regarding the existence of super-exponentially decaying solutions, of our main results in the theory of dynamical systems are presented as well.

In particular, we will present detailed results if the operator under study is a finite rank perturbation of a Volterra operator. The associated characteristic matrix (in the sense of [1] and [2]) allows a detailed spectral analysis of the operator and an explicit computation of the resolvent operator so that the necessary and sufficient conditions for completeness can be verified explicitly.

As an application, we study the spectral properties of the monodromy operator of a periodic functional differential equations in a fairly general setting generalizing recent results in [3].

This lecture is based on joint work with Rien Kaashoek [2].

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Generalization of pairwise models to non-Markovian epidemics on networks

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In this talk, a generalization of pairwise models to non-Markovian epidemics on networks is presented. For the case of infectious periods of fixed length, the resulting pairwise model is a system of delay differential equations (DDEs), which shows excellent agreement with results based on stochastic simulations. Furthermore, we show a new $\mathcal{R}_0$-like threshold quantity and an analytical relation between this and the final epidemic size. In addition, we show that the pairwise model and the analytic results can be generalized to any distribution of the infectious period, using DDEs, and this is illustrated by presenting a general expression for the final epidemic size. By showing the rigorous link between non-Markovian dynamics and pairwise DDEs, we provide the framework for a deeper and more systematic understanding of the impact of non-Markovian dynamics on threshold quantities and final epidemic size. This is a joint work with Gergely Röst and István Z. Kiss.

State-dependent delays which yield complicated motion

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The lecture deals with the construction of a state-dependent delay $d = d(\phi) \in (0, 2)$, with $d(\phi) = 1$ for small $\phi \in C^1 = C^1([-2, 0])$, so that the equation

$$x'(t) = -\alpha\, x(t - d(x_t))$$

with α close to $\frac{5\pi}{2}$ has a solution which is homoclinic to zero, and the linearized solution operators along the homoclinic loop in the solution manifold

$$\{\phi \in C^1 : \phi'(0) = -\alpha\, \phi(-d(\phi))\}$$

act as isomorphisms on a leading eigenspace. The result is part of joint work with Bernhard Lani-Wayda [1] which establishes shift dynamics close to the homoclinic loop. - Another construction yields state-dependent delays so that the said regularity property of the linearized solution operators is violated [2].

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[2] H.-O. Walther, Merging homoclinic solutions due to state-dependent delay, DOI 10.1016/j.jde.2015.02.009, *J. Differential Equations*, to appear.

The Fisher–KPP equation with moving favorable and lethal habitats

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We consider the Fisher–KPP equation in a wave-like shifting environment for which the wave profile of the environment is given by a monotonically decreasing function changing signs (shifting from favourable to unfavourable environment). This type of equation arises naturally from the consideration of infection spread in a classical SIS epidemiological model of a host population where the disease impact on host mobility and mortality is limited. We conclude that there are three different ranges of the disease transmission rate where the disease spread has distinguished spatiotemporal patterns: extinction; spread (as KPP or pulse waves) in pace with the host invasion; spread not in a wave format and slower than the host invasion. This talk is based on a joint work with Y. Lou and J. Fang.

Delay-induced patterns in a two-dimensional lattice of coupled oscillators

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We show how a variety of stable spatio-temporal periodic patterns can be created in 2D-lattices of coupled oscillators with non-homogeneous coupling delays. The results are illustrated using the FitzHugh–Nagumo coupled neurons as well as coupled limit cycle (Stuart–Landau) oscillators. A "hybrid dispersion relation" is introduced, which describes the stability of the patterns in spatially extended systems with large time-delay (see more details `http://www.nature.com/srep/2015/150217/srep08522/full/srep08522.html`).