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Bicircular projections and characterization of Hilbert spaces. (English)

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The present paper contains some results about JB^*-triples. JB^*-triples are a generalization of operator algebras. For example, all C^*-algebras and JB-algebras are examples of JB^*-triples. The authors state that their research described in the paper was motivated by the investigation of the same authors on continuous Reinhardt domains. These are open subsets of $C_0(\Omega)$ satisfying certain extra conditions. In the study of Reinhardt domains, one is led to showing that the kernel of certain projections with one-dimensional range and an additional norm property, which is called bicircularity, are Hilbert spaces. The authors notice as well that the notion of bicircularity is purely a Banach space notion in nature. In fact, if X is a complex Banach space in some norm and P is a bounded linear projection from X on X, then P is called bicircular if the mappings of the form $e^{i\alpha}P + e^{i\beta}\overline{P}$, where $\overline{P} = \mathbf{1} - P$, are isometric for all $\alpha, \beta \in \mathbb{R}$. It is quite natural to consider a problem of description of the bicircular projections of a given Banach space. One can easily see that the answer depends on the norm of the space and can be different if an equivalent norm is taken instead.
In the present paper, the authors study this type of projections in the classical spaces $B(H)$, $S(H)$ and $A(H)$ consisting, respectively, of all bounded linear, all symmetric bounded linear and all antisymmetric bounded linear operators $X : H \to H$, where H is a complex Hilbert space. So, what for them was at first a technical issue in the study of Reinhardt domains has become an independent, algebraic in nature, research, which is that of JB^*-triples. The authors prove here that the bicircular projections of a JB^*-triple X are exactly the skew derivations of X.
The main result of the paper is a characterization of Hilbert spaces in the category of JB^*-triples. The authors obtain that if there are given a JB^*-triple X and a bicircular projection of rank one $P : X \to X$, then there are two closed ideals J and H of X such that $X = J \dotplus H$, where H is isomorphic to a Hilbert space, $P(X) \subset H$ and $J \subset \ker(P)$.

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Keywords : Reinhardt domains; JB^*-triples; bicurcular projections; skew derivations; Hilbert spaces
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