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Zeroes of Schrödinger eigenfunctions at potential singularities. (English)

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The note contains the proof of the following proposition. Let Ω be a domain in $\mathbb{R}^N$ (N$\geq$ 3) and $u \in C(\Omega)$ be a function satisfying in distribution sense the Schrödinger equation $-\Delta u + qu = \lambda u$ with $0 \leq q \in L^1_{loc}(\Omega)$, $\lambda \in \mathbb{R}$. Then $u(0) = 0$ unless $\overline{\lim}_{r\downarrow 0} \quad r^{-N+2} \int_{|x|<r} q(x)dx < \infty$. In particular, if $q(x) \geq \rho|x|^{-(2+\alpha)}$, $x \in \Omega$, $\rho, \alpha > 0$ then every continuous solution u vanishes at 0.

This proposition is a generalization allowing an a bit stronger singularity of q(x) at the origin then the result by *S. H. Alimov* and *I. Yoo'* [Acta Sci. Math. (to appear)] who proved the same property of u for $q = q_0 + q_1 \geq 0$, $q_0 \in \mathcal{L}^2(\Omega)$ is a radially symmetric function, $q_0 \leq C|x|^{-2}$, $q_1 \in \mathcal{L}^\infty(\Omega)$.

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Keywords : zeroes of the eigensolutions; singular potentials; Schrödinger equation; singularity

Classification :

∗35J10 Schroedinger operator
35P05 General spectral theory of PDE
35B05 General behavior of solutions of PDE