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On the manifold of complemented principal inner ideals in JB*-triples. (English summary)
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A JB*-triple Z is a Banach space with a continuous triple product $\{x,y,z\}$ that is symmetric bilinear in x,z, conjugate linear in y, satisfies the C^*-axiom $\|\{x,x,x\}\| = \|x\|^3$, and is such that, for the operator $D(a,b)\colon x\to\{a,b,x\}$ where $D(a):=D(a,a)$, $D(a)$ is Hermitian and positive and $iD(a)$ is a derivation. C^*-algebras are JB*-triples with the product $\{x,y,z\}=(xy^*z+zy^*x)/2$. Perhaps the most important reason that this generalization of C^*-algebras is studied is because the unit balls JB*-triples are exactly the symmetric Hermitian complex Banach manifolds of non-compact type.

Less well known is the fact that JB*-triples can be used to construct complex symmetric Hermitian manifolds of compact type. As shown by Kaup, one such construction is the family $\mathbb{P}$ of all complemented principal inner ideals J_w of a JB*-triple Z as a submanifold of the Grassmannian of all complemented subspaces of Z. Since principal inner ideals are complemented if and only if they are generated by a tripotent $v=\{v,v,v\}$, one can study the holomorphic structure of $\mathbb{P}$ in terms of $\mathbb{M}$, the manifold of Neher equivalence classes of the set of tripotents M (where two tripotents e,f are equivalent if they generate the same principal inner ideal $J_e=J_f$). Equivalently, one can also study $\mathbb{P}$ via $\mathbb{D}$, the manifold of derivations $\{iD(e)\}$, since two tripotents e and f are equivalent if and only if $D(e)=D(f)$. The purpose of the present paper is to exploit the holomorphic and topological relationships of $\mathbb{M}$, $\mathbb{P}$, and $\mathbb{D}$.

The first main result of the paper (Theorem 3.2) is the algebraic construction of the tangent space at any point $iD(e)$ in the real analytic manifold $\mathbb{D}$ as the space of certain algebraically defined operators $K(e,u)$ (see equation(1)) where u lies in the subspace $Z_{1/2}(e)$. Note that every tripotent e gives rise to a Peirce decomposition $Z_1(e)+Z_{1/2}(e)+Z_0(e)$ (see preliminaries). Since the derivations of Z are well known to be a Banach-Lie algebra whose Lie group is the set of linear algebraic homomorphisms of Z, the authors then can use the exponential to generate a real analytic atlas for $\mathbb{M}$.

In Section 4, the authors introduce and obtain results about a real analytic auxiliary manifold which is the subset of $Z\times Z$ consisting of the union of sets $e\times Z_1(e)$ where e is a tripotent and $Z_1(e)$ is its associated inner ideal. This is used to prove the main result, Theorem 6.1, that for each tripotent $e\in M$, there is a neighbourhood W of 0 in $Z_{1/2}(e)$ and a real analytic map $Y_e\colon W\to M$ such that $Y_e(0)=e$ and $(\exp D(u,e))J_e=J_{Y_e(u)}$. It follows in Corollary 6.3 that the Lie algebra of smooth $\sim$-equivariant vector fields on M (defined and characterized by Peirce projections in Section 5) is isomorphic to the Lie algebra of smooth vector fields on $\mathbb{P}$. In the special case when the JB*-triple can be isomorphically (and hence isometrically) represented in $\mathcal{L}(H)$ via the product $(xy^*z+zy^*x)/2$, the very interesting Theorem 6.4 then gives a canonical holomorphic atlas for $\mathbb{P}$ of the form $\{T_e\colon e\in M\}$, where $T_e(u)=s(\exp D(e,u)e)$ for $u\in Z_{1/2}(e)$. Here $s(x)$ stands for the support tripotent of x. It is conjectured that this result holds generally.

Using these tools, the authors can analyze the topology of $\mathbb{P}$ in terms of several new results (in sections 3 and 5) about the Hausdorff distance between elements of $\mathbb{M}$ and operator distance on $\mathbb{D}$. Finally, in Section 7, the authors relate the holomorphic structure of $\mathbb{P}$ to the holomorphic structure of Z via Proposition 7.2. This states that for every open subset $\mathcal{U}$ of $\mathbb{M}$, a function $\Phi\colon \mathcal{U} \to B$ into a Banach space B is holomorphic if and only if for every $e \in U := \pi^{-1}\mathcal{U}$ there exists an open subset V of Z with $e \in V$ along with a holomorphic function $\varphi\colon V \to B$ such that $\varphi(f) = \Phi(\pi(f))$ whenever $f \in U \cap V$. (Here π is the canonical map from M to $\mathbb{M}$.)

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.