MR2002716 (2004e:32001) 32A07 (46G20 46K70)
Stachó, L. L. (H-SZEG-B); **Zalar, B. [Zalar, Borut]** (SV-MAR)
Symmetric continuous Reinhardt domains. (English summary)
Arch. Math. (Basel) **81** (2003), *no. 1,* 50–61.

A classical Reinhardt domain is an open connected subset D of $\mathbb{C}^n$ which is invariant under the transformations $M_{\lambda_1,\dots,\lambda_n}\colon(z_1,\dots,z_n)\mapsto(\lambda_1 z_1,\dots,\lambda_n z_n)$ with $|\lambda_1|=1,\dots,|\lambda_n|=1$. The Reinhardt domain D is said to be complete if $M_{\lambda_1,\dots,\lambda_n}D\subset D$ whenever $|\lambda_1|\le 1,\dots,|\lambda_n|\le 1$. Reinhardt domains with $0\in D$ were studied by T. Sunada [Proc. Japan Acad. **50** (1974), 119–123; MR0414915 (54 #3007)], who classified them from the point of view of biholomorphic equivalence. He proved that holomorphically symmetric Reinhardt domains are (up to a linear isomorphism) a direct product of Euclidean balls.

In the classical definition of a Reinhardt domain, the n-tuples of complex numbers can be viewed as functions from the discrete topological space $\Omega:=\{1,2,\dots,n\}$ to $\mathbb{C}^n$, and a Reinhardt domain $D\subset\mathcal{C}(\Omega)$ is complete if

$$(f\in D,\ g\in\mathcal{C}(\Omega),\ |g|\le|f|)\Longrightarrow g\in D.$$

This definition directly extends to the lattice $\mathcal{C}_0(\Omega)$ of all continuous complex valued functions that vanish at infinity on Ω, giving rise to the notion of continuous Reinhardt domains. The authors study bounded holomorphically symmetric continuous Reinhardt domains in the above sense. They prove that every such domain D can be represented in the form

$$D=\{f\in\mathcal{C}_0(\Omega)\colon \sup_{j\in J}\sum_{\omega\in\Omega_j} m(\omega)|f(\omega)|^2<1\},$$

where $\{\Omega_j\colon j\in J\}$ is a partition of the space Ω such that the cardinalities of the Ω_j are finite and bounded from above and $m\colon\Omega\to\mathbb{R}^+$ is a function whose values are positive real numbers. In other words, D is exactly the open unit ball of an l_∞-direct sum of a family of finite-dimensional Hilbert spaces H_j whose dimensions are precisely the cardinalities of Ω_j $(j\in J)$, these dimensions being bounded from above by a constant M which is a certain geometric characteristic of D. This result clearly extends those of Sunada as well as those of J.-P. Vigué [Ark. Mat. **36** (1998), no. 1, 177–190; MR1611169 (99b:58018)] relative to continuous products of discs of variable radius.

The arguments in the paper have a neat Jordan theoretical nature and the key point is a remarkable extension of the classical Banach-Stone theorem on the isometries of the space $\mathcal{C}_0(\Omega)$. In fact this theorem corresponds to the case in which the above partition consists of singletons $\Omega_j=\{\omega_j\}$ and the weight function m is the constant 1. According to the classical Banach-Stone theorem, the set of extreme points of the unit ball of the dual space of $\mathcal{C}_0(\Omega)$ is precisely the set

$$\{\lambda\delta_\omega(\cdot)\colon\ \omega\in\Omega,\ |\lambda|=1\}$$

of unitary multiples of the one-point evaluation measures on Ω. In the case discussed by the authors, the space $\mathcal{C}_0(\Omega)$ carries its norm as the JB*-triple associated to the domain D, which is equivalent to the usual norm of the supremum over Ω, and each extreme point of the unit dual ball of $\mathcal{C}_0(\Omega)$ is precisely the unit sphere of a finite-dimensional complex Hilbert space H_j, the dimension of which is the cardinality of Ω_j.

Reviewed by *J. M. Isidro*

References

1. R. Braun, W. Kaup and H. Upmeier, On the automorphisms of symmetric and Reinhardt domains in complex Banach spaces. Manuscripta Math. **25**, 97–133 (1978). MR0500878 (80g:32003)
2. T. J. Barton and R. M. Timoney, Weak^*-continuity of Jordan triple products and applications. Math. Scand. **59**, 177–191 (1986). MR0884654 (88d:46129)
3. S. Dineen, Complete holomorphic vector fields on the second dual of a Banach space. Math. Scand. **59**, 131–142 (1986). MR0873493 (88h:32029)
4. Y. Friedman and B. Russo, Structure of the predual of a JBW^*-triple. J. Reine Angew. Math. **356**, 67–89 (1985). MR0779376 (86f:46073)
5. E. Hewitt and K. Strømberg, Real and Abstract Analysis. Berlin 1965. MR0188387 (32 #5826)
6. J. M. Isidro and L. L. Stachó, Holomorphic Automorphism Groups in Banach Spaces. North Holland Math. Stud. **105**, Elsevier 1984. MR0779821 (86f:32037)
7. W. Kaup, A Riemann mapping theorem for bounded symmetric domains in complex Banach spaces. Math. Z. **138**, 503–509 (1983). MR0710768 (85c:46040)
8. K. McCrimmon, Compatible Peierce decompositions of Jordan triple systems. Pacific J. Math. **103**, 57–102 (1982). MR0687964 (84e:17018)
9. E. Neher, Jordan Triple Systems by the Grid Approach. LNM **1280**, Berlin-Heidelberg-New York 1987. MR0911879 (89b:17024)
10. B. Russo, Structure of JB^*-triples. In: Jordan Algebras, W. Kaup, K. McCrimmon and H. P. Petersson eds., Berlin 1994. MR1293321 (95h:46109)
11. T. Sunada, On bounded Reinhardt domains. Proc. Japan Acad. **50**, 119–123 (1974). MR0414915 (54 #3007)
12. J.-P. Vigué, Automorphismes analytiques des domaines produits. Ark. Mat. **36**, 177–190 (1998). MR1611169 (99b:58018)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.