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Holomorphic automorphisms of continuous products of balls. (English summary)
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The authors consider "continuous products" $\mathbf{D}$ of bounded domains D_ω in a Banach space E indexed by $\omega \in \Omega\ =$ a compact Hausdorff space. One result establishes conditions on D_ω so that every complete holomorphic vector field on the product is "fibre preserving".

For the case where each D_ω is the unit ball for some equivalent norm on E, the orbit of the origin under biholomorphic automorphisms of $\mathbf{D}$ is characterised via the corresponding orbits in each D_ω and a continuity condition for the coordinate-wise triple product with the middle variable fixed. This result can be applied to give a simple form for the case when D_ω is independent of ω.

Another case investigated is where there is a family of bounded surjective linear isomorphisms φ_ω of E, norm bounded above and with inverse bounded above in norm, so that D_ω is the image of the unit ball of E under φ_ω. Assume $(x,\omega) \in E \times \Omega \mapsto \varphi_\omega(x)$ is upper semicontinuous on the product space, and that the unit ball of E is a symmetric domain. Then the orbit of the origin in $\mathbf{D}$ is shown to contain a part of $\mathbf{D}$ with a vanishing condition at points of discontinuity of $\omega \in \Omega \mapsto \varphi_\omega$ (for the strong operator topology on linear self-maps of E). In an example of a product of two-dimensional Hilbert balls, the containment is strict. In the case where each φ_ω is a positive multiple of the identity, the vanishing condition gives the precise orbit (recovering a result of J.-P. Vigué [Ark. Mat. **36** (1998), no. 1, 177–190; MR1611169 (99b:58018)]).

Reviewed by *Richard M. Timoney*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.