MR1273340 (95f:46094) 46K70 (46L70)
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Weakly continuous JB*-triples.
Math. Nachr. 166 (1994), 305-315.
Let $E$ be a JB*-triple; for details see a paper by Kaup [Math. Z. 183 (1983), no. 4, 503-529; MR0710768 (85c:46040)]. JB*-triples are natural generalizations of JB*-algebras (a Jordan version of $C^{*}$-algebras) and are characterized by a certain ternary product $(x, y, z) \mapsto\{x y z\}$ on $E$. For instance, if $E$ is a $C^{*}$-algebra this ternary product is given by $\{x y z\}=\left(x y^{*} z+z y^{*} x\right) / 2$. Denote by $\operatorname{Cont}_{\mathbf{w}}(E)$ the space of all $a \in E$ such that the squaring map $z \mapsto\{z a z\}$ is weakly continuous. The main results of this paper are the following theorems.
Theorem: Let $E$ be a commutative JB*-triple and $\pi: S \rightarrow \Omega$ the corresponding principal T-fibre bundle realization of $E$ as $E=\left\{f \in \mathcal{C}_{0}(S): f(t s)=t f(s)\right.$, for all $\left.t \in \mathbf{T}\right\}$. Then $\operatorname{Cont}_{\mathbf{w}}(E)=$ $\left\{f \in E: f \mid \pi^{-1}(\Pi)=0\right\}$ holds with $\Pi$ the maximal perfect subset of the spectrum $\Omega$ of $E$.
Theorem: A JB*-triple is weakly continuous $\left(E=\operatorname{Cont}_{\mathbf{w}}(E)\right)$ if and only if the following three conditions are satisfied: (1) $E$ has the dual RNP (i.e. the dual of $E$ has Radon-Nikodým property); (2) every $w^{*}$-dense representation from $E$ to a Cartan factor is elementary; (3) every spin factor representation of $E$ is finite-dimensional.

Reviewed by Josuke Hakeda
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