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Convex Bodies: The Brunn–Minkowski Theory

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PREFACE

The Brunn–Minkowski theory is the classical core of the geometry of convex bodies. It originated with the thesis of Hermann Brunn in 1887 and is in its essential parts the creation of Hermann Minkowski, around the turn of the century. The well-known survey of Bonnesen and Fenchel in 1934 collected what was already an impressive body of results, though important developments were still to come, through the work of A.D. Aleksandrov and others in the thirties. In recent decades, the theory of convex bodies has expanded considerably; new topics have been developed and originally neglected branches of the subject have gained in interest. For instance, the combinatorial aspects, the theory of convex polytopes and the local theory of Banach spaces attract particular attention now. Nevertheless, the Brunn–Minkowski theory has remained of constant interest owing to its various new applications, its connections with other fields, and the challenge of some resistant open problems.

Aiming at a brief characterization of Brunn–Minkowski theory, one might say that it is the result of merging two elementary notions for point sets in Euclidean space: vector addition and volume. The vector addition of convex bodies, usually called Minkowski addition, has many facets of independent geometric interest. Combined with volume, it leads to the fundamental Brunn–Minkowski inequality and the notion of mixed volumes. The latter satisfy a series of inequalities which, due to their flexibility, solve many extremal problems and yield several uniqueness results. Looking at mixed volumes from a local point of view, one is led to mixed area measures. Quermassintegrals, or Minkowski functionals, and their local versions, surface area measures and curvature measures, are a special case of mixed volumes and mixed area

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Section references are given. Most of the references occur in the section notes.

Proof. From (4.3.5) and the assumption $s(K) = s(K')$ we obtain

$$|h(K, u) - h(K', u)| \leqq \int_{S^{n-1}} |g_n(\langle u, v \rangle)| \, d\nu(v),$$

where ν is the variation of the signed measure $S_1(K, \cdot) - S_1(K', \cdot)$. The assumption (4.3.7) yields $\nu(\omega) \leqq \varepsilon \mathcal{H}^{n-1}(\omega)$. The existence of the constant c_n now follows from (4.3.6). ■

Notes for Section 4.3

1. *Christoffel's problem.* It is clear from the Introduction in Christoffel (1865) that the surfaces that he studied are assumed to be convex ('allenthalben gewölbt'), but in the existence part of his investigation no such property is discussed (an erroneous statement to the contrary was made in Bonnesen & Fenchel [8], p. 123). Favard (1933b) and Süss (1932b, 1933) claimed to prove that the condition $\int u \, d\varphi(u) = o$ is sufficient for the existence of a convex body K with $S_1(K, \cdot) = \varphi$ (under smoothness assumptions). However, Aleksandrov (1937c, 1938a) gave examples of positive measures on $\mathcal{B}(S^{n-1})$, some even with analytic densities, which satisfy that necessary condition and are not the first area measure $S_1(K, \cdot)$ of any convex body K. A sufficient, but not necessary, condition was found by Pogorelov (1953).

   The proof of the uniqueness result, Theorem 4.3.1, by means of spherical harmonics goes back essentially to Hurwitz (1902). His proof for the three-dimensional case is reproduced in Blaschke (1924), §95, and Pogorelov [27], and was extended to higher dimensions by Kubota (1925b).

   A brief review of the various treatments of Christoffel's problem was given by Firey (1981). An application of Firey's (1967b) existence result to the study of surfaces of constant width appears in Fillmore (1969).

   In the centrally symmetric case, Goodey & Weil (1990+) found a connection between Christoffel's problem and Radon transforms.
2. *Christoffel's problem for polytopes.* The necessary and sufficient conditions for first-order area measures found by Firey and Berg are not easy to handle in concrete cases. For instance, it seems difficult to derive from these criteria a description of the first-order area measures of polytopes. However, Christoffel's problem for polytopes can be given an independent treatment by direct elementary methods. This has been done by Schneider (1977a).
3. *The length measure.* In the plane, the map $K \mapsto S_1(K, \cdot)$ establishes an isomorphism between the convex cone of convex bodies with Steiner point o and the cone of measures φ on $\mathcal{B}(S^{n-1})$ with $\int u \, d\varphi(u) = o$. This fact is sometimes useful for the treatment of decompositions of plane convex bodies. Schneider (1974e) used the length measure in the investigation of the asymmetry classes of convex bodies. Kallay (1975) characterized the extreme convex sets K in the set of bodies in $\mathcal{K}^2$ with a given width function in terms of a property of the Radon–Nikodym derivative of $S_1(K, \cdot)$.

   A detailed study of the length measure was also made by Letac (1983).
4. *Stability results.* Theorem 4.3.5, for the special case of three-dimensional convex bodies with twice continuously differentiable support functions, is due to Pogorelov [27], p. 502. The general case is in Schneider [43], Theorem (9.8).

Sen'kin (1966a,b) proved the estimate

$$|w(K, u) - w(K', u)| \leqq \frac{1}{\pi} \sup_{\omega \subset S^2} |S_1(K, \omega) - S_1(K', \omega)|$$

for the width $w(\cdot, u)$ in any direction $u \in S^2$, for convex bodies $K, K' \in \mathcal{K}^3$. He used an elementary argument for strongly isomorphic polytopes and then Aleksandrov's approximation theorem 2.4.14.

4.4. Additive extension

The generalized curvature measure $\Theta_m(K, \cdot)$ is an additive function of K, by Theorem 4.2.1. In Section 3.4 we introduced the convex ring $U(\mathcal{K}^n)$, consisting of all finite unions of convex bodies in $\mathbb{E}^n$, and collected some results about the additive extension of valuations from $\mathcal{K}^n$ to $U(\mathcal{K}^n)$. In the present section we shall treat the additive extension of the generalized curvature measures Θ_m to the convex ring. While the existence of such an extension could be deduced from general results on valuations (see Section 3.4, Note 7), we prefer to give an explicit construction that provides additional insight and at the same time extends the Steiner formula to local 'parallel sets with multiplicity'. The availability of curvature measures for sets more general than convex bodies is useful for applications, and the additivity of the extension permits the almost automatic generalization of some integral-geometric formulae to be proved in the next section.

First let $K \in \mathcal{K}^n$ be a convex body, let $\rho > 0$ and $\eta \in \mathcal{B}(\Sigma)$. In Section 4.1 we defined the local parallel set $M_\rho(K, \eta)$, and equation (4.2.4) states that

$$\mathcal{H}^n(M_\rho(K, \eta)) = \frac{1}{n} \sum_{m=0}^{n-1} \rho^{n-m} \binom{n}{m} \Theta_m(K, \eta).$$

In order to obtain a Steiner formula such as the above for non-convex sets $K \in U(\mathcal{K}^n)$ also, we consider, instead of the parallel set $M_\rho(K, \eta)$, its characteristic function $c_\rho(K, \eta, \cdot)$ and extend this additively to $U(\mathcal{K}^n)$. The value $c_\rho(K, \eta, \cdot)$ is then interpreted as counting the multiplicity with which the point x belongs to the parallel set of K with respect to η.

Recall that the Euler characteristic χ is the (unique) valuation on the convex ring $U(\mathcal{K}^n)$ satisfying $\chi(K) = 1$ for $K \in \mathcal{K}^n$; see Theorem 3.4.12. Now, for $K \in U(\mathcal{K}^n)$ and points $q, x \in \mathbb{E}^n$ we define the *index of K at q with respect to x* by

$$j(K, q, x) := \begin{cases} 1 - \lim_{\delta \downarrow 0} \lim_{\varepsilon \downarrow 0} \chi(K \cap B(x, |x - q| - \varepsilon) \cap B(q, \delta)) & \text{if } q \in K, \\ 0 & \text{if } q \notin K. \end{cases}$$

If K is convex, then clearly

$$j(K, q, x) = \begin{cases} 1 & \text{if } q = p(K, x), \\ 0 & \text{otherwise.} \end{cases} \tag{4.4.1}$$

To prove the existence of the limits, choose a representation $K = \bigcup_{i=1}^{r} K_i$ with $K_i \in \mathcal{K}^n$. Without loss of generality, we may assume that $q \in K_i$ for $i = 1, \ldots, m$ and $q \notin K_i$ for $i > m$, where $1 \leqq m \leqq r$. We can choose $\delta_0 > 0$ such that $K_i \cap B(q, \delta) = \emptyset$ for $i \in \{m+1, \ldots, r\}$ and $0 < \delta < \delta_0$. If $\delta < \delta_0$ is fixed, then for all sufficiently small $\varepsilon > 0$ the inequality

$$K_v \cap B(x, |x-q| - \varepsilon) \cap B(q, \delta) \neq \emptyset$$

holds for all $v \in S(m)$ with $j(K_v, q, x) = 0$ (where we use the notation introduced in Section 3.4). In fact, for $v \in S(m)$ we have $q \in K_v$, and if $j(K_v, q, x) = 0$, then (4.4.1) implies that K_v and hence $K_v \cap B(q, \delta)$ contains a point whose distance from x is smaller than that of q. For sufficiently small $\varepsilon > 0$ we thus have

$$j(K_v, q, x) = 1 - \chi(K_v \cap B(x, |x-q| - \varepsilon) \cap B(q, \delta)).$$

Using the additivity of χ, we deduce that

$$\begin{aligned} 1 - \chi(K \cap B(x, |x-q| - \varepsilon) \cap B(q, \delta)) \\ &= \sum_{v \in S(m)} (-1)^{|v|-1} [1 - \chi(K_v \cap B(x, |x-q| - \varepsilon) \cap B(q, \delta))] \\ &= \sum_{v \in S(m)} (-1)^{|v|-1} j(K_v, q, x) \\ &= \sum_{v \in S(r)} (-1)^{|v|-1} j(K_v, q, x). \end{aligned}$$

The right-hand side does not depend on ε or δ, which shows the existence of the limits.

The index function j thus defined is additive in its first argument, that is, for fixed $q, x \in \mathbb{E}^n$ and for $K, L \in U(\mathcal{K}^n)$ we have

$$j(K \cup L, q, x) + j(K \cap L, q, x) = j(K, q, x) + j(L, q, x).$$

This follows from the definition; in fact, for $q \notin K \cup L$ both sides are zero and for $q \in K \cap L$ it follows from the additivity of χ. If $q \in K \setminus L$ (and similarly for $q \in L \setminus K$) one has to observe that $(K \cup L) \cap B(q, \delta) = K \cap B(q, \delta)$ for all sufficiently small $\delta > 0$.

Now for $K \in U(\mathcal{K}^n)$, $\eta \in \mathcal{B}(\Sigma)$, $\rho > 0$ and $x \in \mathbb{E}^n$ we define

$$c_\rho(K, \eta, x) := \sum_{\substack{q \in \mathbb{E}^n \setminus \{x\} \\ (q, \overline{x-q}) \in \eta}} j(K \cap B(x, \rho), q, x)$$

where $\overline{x-q} := (x-q)/|x-q|$. Actually, the sum is finite, since $j(K, q, x) \neq 0$ for $K = \bigcup_{i=1}^{r} K_i$ with $K_i \in \mathcal{K}^n$ implies, by the additivity of $j(\cdot, q, x)$, that there is some $v \in S(r)$ for which $j(K_v, q, x) \neq 0$ and hence $q = p(K_v, x)$.

If K is convex, then (4.4.1) obviously implies that

$$c_\rho(K, \eta, x) = \begin{cases} 1 & \text{if } 0 < d(K, x) \leqq \rho \text{ and } (p(K, x), u(K, x)) \in \eta, \\ 0 & \text{otherwise.} \end{cases}$$

Thus $c_\rho(K, \eta, \cdot)$ is the characteristic function of the local parallel set $M_\rho(K, \eta)$. From the additivity of $j(\cdot, q, x)$ it follows that $c_\rho(\cdot, \eta, x)$ is additive on $U(\mathcal{K}^n)$. Hence, for $K = \bigcup_{i=1}^{r} K_i$ with $K_i \in \mathcal{K}^n$ we have

$$c_\rho(K, \eta, \cdot) = \sum_{v \in S(r)} (-1)^{|v|-1} c_\rho(K_v, \eta, \cdot).$$

Since the right-hand side is a finite sum of integrable functions, we can define

$$\mu_\rho(K, \eta) := \int_{\mathbb{E}^n} c_\rho(K, \eta, x) \, d\mathcal{H}^n(x)$$

for $K \in U(\mathcal{K}^n)$ and $\eta \in \mathcal{B}(\Sigma)$. The notation is consistent with that of Section 4.1, and $\mu_\rho(\cdot, \eta)$ is an additive function on $U(\mathcal{K}^n)$. Hence, for $K = \bigcup_{i=1}^{r} K_i$ with $K_i \in \mathcal{K}^n$ we have

$$\begin{aligned} \mu_\rho(K, \eta) &= \sum_{v \in S(r)} (-1)^{|v|-1} \mu_\rho(K_v, \eta) \\ &= \sum_{v \in S(r)} (-1)^{|v|-1} \frac{1}{n} \sum_{m=1}^{n-1} \rho^{n-m} \binom{n}{m} \Theta_m(K_v, \eta) \\ &= \frac{1}{n} \sum_{m=0}^{n-1} \rho^{n-m} \binom{n}{m} \sum_{v \in S(r)} (-1)^{|v|-1} \Theta_m(K_v, \eta). \end{aligned}$$

Since the left-hand side depends only on the point set K and not on its chosen representation as a finite union of convex bodies, the same is true for the coefficients of the polynomial in ρ on the right-hand side. Hence, we are now in a position to define

$$\Theta_m(K, \eta) := \sum_{v \in S(r)} (-1)^{|v|-1} \Theta_m(K_v, \eta)$$

for $m = 0, \ldots, n-1$. Thus $\Theta_m(K, \cdot)$ is a finite signed measure on $\mathcal{B}(\Sigma)$. From this representation (or from the additivity of $\mu_\rho(\cdot, \eta)$) we deduce that $\Theta_m(\cdot, \eta)$ is additive on $U(\mathcal{K}^n)$. In this way, we obtain the (unique) additive extension of the generalized curvature measures to the convex ring, and for the generalized parallel volume $\mu_\rho(K, \eta)$ we arrive at the Steiner formula

$$\mu_\rho(K, \eta) = \frac{1}{n} \sum_{m=0}^{n-1} \rho^{n-m} \binom{n}{m} \Theta_m(K, \eta).$$

As in Section 4.2, we specialize the generalized curvature measures by putting

$$C_m(K, \beta) = \frac{n\kappa_{n-m}}{\binom{n}{m}} \Phi_m(K, \beta) = \Theta_m(K, \beta \times S^{n-1}),$$

$$S_m(K, \omega) = \frac{n\kappa_{n-m}}{\binom{n}{m}} \Psi_m(K, \omega) = \Theta_m(K, \mathbb{E}^n \times \omega)$$

for $K \in U(\mathcal{K}^n)$, Borel sets $\beta \in \mathcal{B}(\mathbb{E}^n)$, $\omega \in \mathcal{B}(S^{n-1})$ and for $m = 0, \dots, n-1$; further

$$\Phi_n(K, \beta) := \mathcal{H}^n(K \cap \beta).$$

Of these additive extensions of the curvature measures and area measures to the convex ring, it will mainly be the signed measures C_m (or Φ_m) that will be used in the sequel. We shall first extend the interpretation of C_{n-1} given by (4.2.23).

Theorem 4.4.1. *If $K \in U(\mathcal{K}^n)$ is the closure of its interior, then*

$$C_{n-1}(K, \beta) = \mathcal{H}^{n-1}(\beta \cap \operatorname{bd} K)$$

for $\beta \in \mathcal{B}(\mathbb{E}^n)$.

Proof. Let $K = \bigcup_{i=1}^r K_i$ with $K_i \in \mathcal{K}^n$. If among the convex bodies $K_1, \dots, K_r$ there were one of dimension less than n not covered by the union of the others, then K would not be the closure of its interior. Hence, lower-dimensional bodies in the representation may be omitted, and we can assume from the beginning that $\dim K_i = n$ for $i = 1, \dots, r$.

For $L \in \mathcal{K}^n$, let $\varphi(L, \cdot)$ denote the characteristic function of the boundary of L. Let $x \in \operatorname{bd} K$. If, say, $x \in K_i$ precisely for $i = 1, \dots, m$, then $x \in \operatorname{bd} K_v$ for $v \in S(m)$ and hence

$$\sum_{v \in S(r)} (-1)^{|v|-1} \varphi(K_v, x) = \sum_{v \in S(m)} (-1)^{|v|-1} \times 1 = 1.$$

Let $\beta \in \mathcal{B}(\mathbb{E}^n)$ and $v \in S(r)$. Theorem 4.2.5 together with the remark after its proof tells us that $C_{n-1}(K_v, \beta) = \mathcal{H}^{n-1}(\beta \cap \operatorname{bd} K_v)$, provided that $\dim K_v \neq n-1$. If $\dim K_v = n-1$, then $\operatorname{relint} K_v \subset \operatorname{int} K$, hence $C_{n-1}(K_v, \beta \cap \operatorname{bd} K) = 0 = \mathcal{H}^{n-1}(\beta \cap \operatorname{bd} K \cap \operatorname{bd} K_v)$. Further, $C_{n-1}(K, \cdot)$ is concentrated on the boundary of K, since

$c_\rho(K, \beta \times S^{n-1}, \cdot) = c_\rho(K, (\beta \cap \operatorname{bd} K) \times S^{n-1}, \cdot)$ by the definition of c_ρ. For $\beta \in \mathcal{B}(\mathbb{E}^n)$ we conclude that

$$\begin{aligned} C_{n-1}(K, \beta) &= C_{n-1}(K, \beta \cap \operatorname{bd} K) \\ &= \sum_{v \in S(r)} (-1)^{|v|-1} C_{n-1}(K_v, \beta \cap \operatorname{bd} K) \\ &= \sum_{v \in S(r)} (-1)^{|v|-1} \mathcal{H}^{n-1}(\beta \cap \operatorname{bd} K \cap \operatorname{bd} K_v) \\ &= \sum_{v \in S(r)} (-1)^{|v|-1} \int_{\beta \cap \operatorname{bd} K} \varphi(K_v, x) \, d\mathcal{H}^{n-1}(x) \\ &= \mathcal{H}^{n-1}(\beta \cap \operatorname{bd} K). \end{aligned}$$

■

For convex bodies $K \in \mathcal{K}^n$, formula (4.2.21) interprets the curvature measure C_0 as the content of the spherical image, namely

$$C_0(K, \beta) = \mathcal{H}^{n-1}(\sigma(K, \beta)).$$

If we want to extend this representation to the elements of the convex ring, we have to count the points of the spherical image with a suitable multiplicity. For this, we use a similar approach to the above and introduce another index function. For $K \in U(\mathcal{K}^n)$, a point $q \in \mathbb{E}^n$ and a vector $u \in S^{n-1}$ we define

$$i(K, q, u) := \begin{cases} 1 - \lim\limits_{\delta \downarrow 0} \lim\limits_{\varepsilon \downarrow 0} \chi(K \cap B(q + (\delta + \varepsilon)u, \delta)) & \text{if } q \in K, \\ 0 & \text{if } q \notin K. \end{cases}$$

If K is convex, then clearly

$$i(K, q, u) = \begin{cases} 1 & \text{if } (q, u) \in \operatorname{Nor} K, \\ 0 & \text{otherwise.} \end{cases} \tag{4.4.2}$$

As for the index function $j(K, q, \cdot)$ one sees that $i(K, q, u)$ is well defined and that $i(\cdot, q, u)$ is an additive function on the convex ring $U(\mathcal{K}^n)$. For $K \in U(\mathcal{K}^n)$, $\beta \in \mathcal{B}(\mathbb{E}^n)$ and $u \in S^{n-1}$ we define

$$c(K, \beta, u) := \sum_{q \in \beta} i(K, q, u).$$

Here we have to allow infinite values, but these can be neglected: Let $K = \bigcup_{i=1}^r K_i$ with $K_i \in \mathcal{K}^n$. If $u \in \bigcap_{i=1}^r \operatorname{regn} K_i$ and $v \in S(r)$, there is at most one point q for which $(q, u) \in \operatorname{Nor} K_v$ and thus $i(K_v, q, u) \neq 0$. It follows from Theorem 2.2.9 that $c(K, \beta, \cdot)$ is finite $\mathcal{H}^{n-1}$-almost everywhere on S^{n-1}. If K is convex and $u \in \operatorname{regn} K$, then (4.4.2) implies that $c(K, \beta, \cdot)$ is the characteristic function of the spherical image

$\sigma(K, \beta)$, hence (4.2.21) can be written as

$$C_0(K, \beta) = \int_{S^{n-1}} c(K, \beta, u)\, d\mathcal{H}^{n-1}(u). \tag{4.4.3}$$

Since both $C_0(\cdot, \beta)$ and $c(\cdot, \beta, u)$ are additive on $U(\mathcal{K}^n)$, the equality extends immediately to sets K of the convex ring. This is the desired interpretation of the curvature measure $C_0(K, \cdot)$ as the $\mathcal{H}^{n-1}$-integral of a multiplicity function for the spherical image.

Finally we remark that

$$\frac{1}{\omega_n} C_0(K, \mathbb{E}^n) = V_0(K) = \chi(K) \tag{4.4.4}$$

for $K \in U(\mathcal{K}^n)$. This is true for convex bodies, and the general case follows by additivity. In the same way, a corresponding result for the index functions is obtained. This is in analogy to well-known index sum formulae for smooth submanifolds. For $K \in U(\mathcal{K}^n)$ and $x \in \mathbb{E}^n \setminus K$ we have

$$\sum_{q \in \mathbb{E}^n} j(K, q, x) = \chi(K), \tag{4.4.5}$$

and if $u \in S^{n-1}$ is regular for K, which means that $u \in \bigcap_{i=1}^{r} \operatorname{regn} K_i$ for some representation $K = \bigcup_{i=1}^{r} K_i$ with $K_i \in \mathcal{K}^n$, we have

$$\sum_{q \in \mathbb{E}^n} i(K, q, u) = \chi(K). \tag{4.4.6}$$

Notes for Section 4.4

1. *Index functions.* The additive extension of the generalized curvature measures to the convex ring by means of an index function was carried out by Schneider (1980a). This had previously been done for the curvature measure C_0 in Schneider (1977d), extending some work of Hadwiger (1969c) and Banchoff (1967) for polyhedra and cell complexes. The index $i(K, q, u)$ is also used in Zähle (1987b).

   As mentioned at the end of Section 4.4, the index functions that were introduced are analogues of index functions known in differential geometry. Calling the point q a critical point of $K \in U(\mathcal{K}^n)$ with respect to the height function $\langle \cdot, u \rangle$ if $i(K, q, u) \neq 0$, one may consider equality (4.4.6) as an analogue of the critical point theorem for smooth submanifolds. Similarly, (4.4.4) is an analogue of the Gauss–Bonnet theorem.

   Suppose that $K \in U(\mathcal{K}^n)$ is the point set of a polyhedral cell complex of which Δ^k is the set of k-dimensional cells. Then the index $i(K, q, u)$ satisfies

$$i(K, q, u) = \sum_{k=0}^{n} (-1)^k \sum_{Z \in \Delta^k} i(Z, q, -u). \tag{4.4.7}$$

   This was proved by Shephard (1968c) for the special case of the boundary complex of a convex polytope, and it can be extended to the general case by means of an argument due to Perles & Sallee (1970). A definition equivalent

   to (4.4.7) was used by Banchoff (1967, 1970) in his investigation of critical point theory, curvature and the Gauss–Bonnet theorem for polyhedra.

2. *Absolute curvature measures on the convex ring.* If the additivity assumption is dropped, other extensions of the curvature measures to the convex ring are possible. Of particular interest are non-negative, or absolute, curvature measures. A non-negative extension of Federer's curvature measures to the convex ring was proposed by Matheron (1975), pp. 119ff. His construction was extended in Schneider (1980a), as follows. For given $K \in U(\mathcal{K}^n)$ and $x \in \mathbb{E}^n$, a point $q \in \mathbb{E}^n$ is called a *projection* of x in K if $q \in K$ and there exists a neighbourhood N of q such that $|x - y| > |x - q|$ for all $y \in K \cap N$, $y \neq q$. The set $\Pi(K, x)$ of all projections of x in K is finite. For $\eta \in \mathcal{B}(\Sigma)$ and $\rho > 0$ let

   $$\bar{c}_\rho(K, \eta, x) := \operatorname{card}\{q \in \Pi(K, x) | 0 < |x - q| \leqq \rho \text{ and } (q, \overline{x - q}) \in \eta\}$$

   and

   $$\bar{\mu}_\rho(K, \eta) := \int_{\mathbb{E}^n} \bar{c}_\rho(K, \eta, x)\, d\mathcal{H}^n(x).$$

   One can show that $\bar{\mu}_\rho(K, \eta)$ satisfies a Steiner formula, that is, a polynomial representation

   $$\bar{\mu}_\rho(K, \eta) = \frac{1}{n} \sum_{m=0}^{n-1} \rho^{n-m} \binom{n}{m} \bar{\Theta}_m(K, \eta),$$

   and that this defines (positive) measures $\bar{\Theta}_0(K, \cdot), \ldots, \bar{\Theta}_{n-1}(K, \cdot)$ on $\mathcal{B}(\Sigma)$, which for $K \in \mathcal{K}^n$ coincide with respectively $\Theta_0(K, \cdot), \ldots, \Theta_{n-1}(K, \cdot)$. The specialization

   $$\bar{C}_m(K, \beta) := \bar{\Theta}_m(K, \beta \times S^{n-1}), \quad \beta \in \mathcal{B}(\mathbb{E}^n),$$

   yields the measures introduced by Matheron.

   The measure $\bar{C}_0(K, \cdot)$ can also be interpreted as follows. For $K \in U(\mathcal{K}^n)$ and $x \in \operatorname{bd} K$, a unit vector $u \in S^{n-1}$ is called a *normal vector* of K at x if there exists a neighbourhood N of x such that $\langle x, u \rangle \geqq \langle y, u \rangle$ for all $y \in K \cap N$. For $\beta \in \mathcal{B}(\mathbb{E}^n)$, let $\bar{c}(K, \beta, u)$ be the number (possibly infinite) of points $x \in K \cap \beta$ for which u is a normal vector. Then

   $$\bar{C}_0(K, \beta) = \int_{S^{n-1}} \bar{c}(K, \beta, u)\, d\mathcal{H}^{n-1}(u).$$

   Thus $\bar{C}_0(K, \cdot)$ can be interpreted as the measure of the 'spherical image with multiplicity'. The normalized total measure $\bar{C}_0(K, \mathbb{E}^n)/\omega_n$ was called the *convexity number* of K by Matheron (1975). It is equal to one for convex bodies, but not only for these.

3. *Other generalizations of curvature measures.* The treatment of curvature measures given in this section was restricted to convex bodies and the sets of the convex ring. Its extension to more general classes of sets is possible, but requires in general deeper methods from geometric measure theory. We give only some brief hints to the literature. Federer's (1959) curvature measures for sets of positive reach were further extended and investigated by Zähle (1984a, b, 1986a, 1987a, b). The extension is to finite unions of sets of positive reach and later to so-called second-order rectifiable sets. Applications to cell complexes and mosaics appear in Zähle (1987c, 1988), Weiss (1986) and Weiss & Zähle (1988). Fu (1989) relates curvature measures to generalized Morse theory.

   The approach to curvature measures via so-called normal cycles, as begun by Wintgen (1982) and Zähle (1986a, 1987a, b), is further extended in work by Fu (1990+a, b). General versions of absolute curvature measures were introduced by Baddeley (1980) and Zähle (1989).

Further different approaches have been followed by Stachó (1979), who constructed generalizations of Federer's curvature measures to arbitrary sets (but with weaker properties), and by Kuiper (1971), who used singular homology to define a sequence of curvature measures, which includes a generalization of C_0.

Curvature measures for (non-convex) polyhedra were considered by Flaherty (1973). A thorough study of curvatures for piecewise linear spaces was made by Cheeger, Müller & Schrader (1984, 1986). The latter paper is also related to some (more general) work of Zähle quoted above. Part of the work of Cheeger, Müller & Schrader can be simplified; see Budach (1989) and also the remark at the end of Section 2 in Schneider (1990c).

Approximation results, of different kinds, for curvature measures appear in Brehm & Kühnel (1982), Cheeger, Müller & Schrader (1984), Lafontaine (1987) and Zähle (1990).

4. *Curvature measures in spaces of constant curvature.* Kohlmann (1988) introduced curvature measures for sets of positive reach in Riemannian space forms. Among other results, he obtained integral representations, Minkowski-type integral formulae (see Section 5.3, Note 3) and characterizations of balls (see Section 7.2, Note 4).

5. *Groemer's extension of the quermassintegrals.* The following extension of the quermassintegrals, which apparently does not generalize to curvature measures, was proposed by Groemer (1972). He introduced a vector space A^n of real functions on $\mathbb{E}^n$ with a pseudonorm such that A^n contains $V(\mathcal{P}^n)$ (see Section 3.4, Note 7) as a proper dense subspace. The elements of A^n are called 'approximable' functions. The system $\mathcal{S}_A$ of subsets of $\mathbb{E}^n$ whose characteristic functions are approximable contains the convex ring $U(\mathcal{K}^n)$ and, for example, the relative interiors of convex bodies. Groemer showed that the quermassintegrals can be extended from $\mathcal{K}^n$ to continuous linear functionals on A^n. In particular, this yields an additive extension of the quermassintegral W_i to the class $\mathcal{S}_A$.

4.5. Integral-geometric formulae

Some integral-geometric formulae can be proved for curvature measures and area measures. Generally speaking, integral geometry is concerned with the computation and application of mean values for geometrically defined functions with respect to invariant measures. In this section we shall treat local versions of some intersection and projection formulae, which in their global versions, namely for the Minkowski functionals, are central results of classical integral geometry. We shall also consider mean value formulae for Minkowski sums, and counterparts to the intersection formulae in the form of kinematic formulae involving the distances between non-intersecting bodies, or between bodies and flats. This yields, among other results, an integral-geometric interpretation of generalized curvature measures.

We assume that the reader is familiar with the topological groups $SO(n)$ of proper rotations of $\mathbb{E}^n$ and G_n of rigid motions of $\mathbb{E}^n$, and with their Haar measures. We normalize the Haar measure ν on $SO(n)$ by imposing that $\nu(SO(n)) = 1$. Every rigid motion $g \in G_n$ determines uniquely a rotation $\rho \in SO(n)$ and a translation vector $t \in \mathbb{E}^n$ so that $gx = \rho x + t$ for $x \in \mathbb{E}^n$, and we write $g = g_{t,\rho}$. The map

$$\gamma: \mathbb{E}^n \times SO(n) \to G_n$$
$$(t, \rho) \mapsto g_{t,\rho}$$

is a homeomorphism (products are always equipped with the product topology). We can define a Haar measure μ on G_n by the image measure

$$\mu := \gamma(\lambda_n \otimes \nu),$$

where λ_n denotes the restriction of $\mathcal{H}^n$ to the Borel sets of $\mathbb{E}^n$; this implies a definite normalization of μ. The measure μ, defined on the σ-algebra $\mathcal{B}(G_n)$ of Borel subsets of G_n, is left invariant and right invariant, thus the motion group is unimodular.

On $A(n, k)$, the set of k-dimensional affine subspaces of $\mathbb{E}^n$, where $k \in \{0, \dots, n\}$, the motion group G_n operates in the natural way, making $A(n, k)$ into a homogenous space. To facilitate the handling of its invariant measure, we choose a k-dimensional linear subspace E_k of $\mathbb{E}^n$ and denote its orthogonal complement by $E_k^\perp$. The map

$$\gamma_k: E_k^\perp \times SO(n) \to A(n, k)$$
$$(t, \rho) \mapsto \rho(E_k + t)$$

is surjective. The usual topology of $A(n, k)$ is the finest topology for which γ_k is continuous. With this topology, $A(n, k)$ is a locally compact, second countable Hausdorff space, the natural operation of G_n on $A(n, k)$ is continuous and $A(n, k)$, with this operation, is a homogeneous G_n-space. The image measure

$$\mu_k := \gamma_k(\lambda_{n-k} \otimes \nu)$$

of the product measure $\lambda_{n-k} \otimes \nu$, where λ_{n-k} is the restriction of $\mathcal{H}^{n-k}$ to the Borel sets of $E_k^\perp$, is a G_n-invariant measure on $A(n, k)$ with a suitable normalization. It does not depend on the special choice of E_k.

In a similar, even simpler, way we could obtain the $SO(n)$-invariant measure ν_k on the compact homogeneous space $G(n, k)$ of k-dimensional linear subspaces of $\mathbb{E}^n$, but we shall not need it in what follows.

Below, we shall sometimes need to know that certain sets of rotations, rigid motions or flats are of measure zero. Some of these facts follow from the results of Section 2.3; a more elementary one is contained in the following lemma.

Two linear subspaces E, F of $\mathbb{E}^n$ are said to be *in special position* if

$$\text{lin}(E \cup F) \neq \mathbb{E}^n \quad \text{and} \quad E \cap F \neq \{o\}.$$

Now let f be the support function of a convex body K of class C^∞_+. Then $\eta(f)>0$ by Corollary 2.5.2. Consider the partial sum

$$f_k := \sum_{m=0}^{k} \sum_{j=1}^{N(n,m)} a_{mj} Y_{mj}$$

of (A.15). Using (A.16) and uniform convergence, we may show that $\eta(f_k) \to \eta(f)$ for $k \to \infty$, hence $\eta(f_k) > 0$ for almost all k. It follows that the partial sum f_k is the restriction of a support function for all sufficiently large k.

Appendix Note

Applications to convex bodies. Spherical harmonics in space and Fourier series in the plane have been applied to a great variety of geometric problems on convex bodies. The first applications of this kind were Hurwitz's (1901) well-known proof of the isoperimetric inequality for closed (not necessarily convex) curves in the plane, the thorough investigation of Hurwitz (1902) in the plane and in three-space, and Minkowski's (1904) characterization of convex bodies of constant width. In the many different later contributions, two main principles of application are dominant. This is either the use of the Parseval relation to obtain quadratic inequalities, or the connection of spherical harmonics with irreducible representations of the rotation group. In particular, this often permits us to solve certain rotation invariant linear functional equations on the sphere by finding the spherical harmonics that satisfy the equation, and a number of geometric questions can be reduced to such equations.

Many authors have made geometric applications of Fourier series and spherical harmonics in the domain of convexity. Among them are Aleksandrov (1937b), Anikonov & Stepanov (1981), Berg (1969b), Blaschke (1914), [5], §23, (1916b, 1923), §90, (1936), Bol (1939), Bourgain (1987, 1991), Bourgain & Lindenstrauss (1988a, b, 1989), Bourgain, Lindenstrauss & Milman (1989), Campi (1981, 1984, 1986, 1988), Chernoff (1969), Cieślak & Góźdź (1987), Dinghas (1939, 1940a), Falconer (1983), Fillmore (1969), Fisher (1987), Focke (1969a), Fuglede (1986b, 1989), Fujiwara (1915, 1919), Fujiwara & Kakeya (1917), Geppert (1937), Gericke (1940a, b, 1941), Goodey & Groemer (1990), Görtler (1937a, b, 1938), Groemer (1990a, 1991a), Hayashi (1924, 1926a, b, c), Inzinger (1949), Kameneckii (1947), Knothe (1957b), Kubota (1918, 1920, 1922, 1925a, b), Matsumura (1930), Meissner (1909, 1918), Minoda (1941), Nakajima (1920), Oishi (1920), Petty (1961a), Rešetnjak (1968a), Sas (1939), Schneider (1967c, 1970a, d, 1971a, b, c, d, 1974a, 1975c, 1977c, 1984, 1989c), Shephard (1968b), Su (1927a, b). For details, see Groemer [49].

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