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SOME TOPOLOGICAL AND MIXED MINIMAX THEOREMS

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Abstract. Some noncompact topological and mixed minimax theorems involving compactly locally upward and finitely weakly downward functions are proved.

1. Introduction

Let X and Y be nonempty sets and let $f : X \times Y \to \mathbf{R}$. A minimax theorem is a theorem that the following equality holds:

$$(*) \qquad \inf_Y \sup_X f(x,y) = \sup_X \inf_Y f(x,y).$$

The usual conditions for a minimax theorem are that f is "convex" in one variable and "concave" in other variables plus certain topological conditions on X and (or) Y and f. The following are some nonlinear concavity-convexity conditions of the function f that have been used in minimax theorems.

(I) Concavity of the function f on X:

(C_*) X is a topological space and Y is a set. For any finite subset A of Y and any r in $\mathbf{R}$, the set

$$\bigcap_{y \in A} \{ x : x \in X, f(x,y) \geqq r \}$$

is connected or empty in X.

(S_*) X is an interval space [22] and Y is a set. For any $x_1, x_2 \in X$ and for all $x \in [x_1, x_2]$,

$$f(x,y) \geqq \min \{ f(x_1,y), f(x_2,y) \}$$

for all $y \in Y$.

Recall that an interval space is a topological space X with a mapping $[\cdot,\cdot] : X \times X \to \{\text{connected subsets of } X\}$ such that $x_1, x_2 \in [x_1, x_2] = [x_2, x_1]$ for all $x_1, x_2 \in X$.

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(D) X and Y are nonempty sets. f is downward [19] on X, that is, for every $\varepsilon > 0$, there exists $\delta > 0$ such that for any $x_1, x_2 \in X$, there exists $x_0 \in X$ such that for all $y \in Y$,

$$f(x_0, y) \geqq \min\{f(x_1, y), f(x_2, y)\} \tag{A}$$

and for all $y \in \{y \in Y : |f(x_1, y) - f(x_2, y)| \geqq \varepsilon\}$,

$$f(x_0, y) \geqq \min\{f(x_1, y), f(x_2, y)\} + \delta. \tag{B}$$

(D_*) X and Y are nonempty sets. f is weakly downward [4, 1, 2] on X, that is, for any $x_1, x_2 \in X$, there exists $x_0 \in X$ such that for all $y \in Y$, (A) holds, and for all $y \in \{y \in Y : f(x_1, y) \neq f(x_2, y)\}$,

$$f(x_0, y) > \min\{f(x_1, y), f(x_2, y)\}. \tag{C}$$

(II) Convexity of the function f on Y:

(C^*) X is a nonempty set and Y is a topological space. For any finite subset A of X and any r in $\mathbf{R}$, the set

$$\bigcap_{x \in A} \{y : y \in Y, f(x, y) \leqq r\}$$

is connected or empty in Y.

(S^*) X is a nonempty set and Y is an interval space. For any $y_1, y_2 \in Y$ and all $y \in [y_1, y_2]$,

$$f(x, y) \leqq \max\{f(x, y_1), f(x, y_2)\} \quad \text{for all} \quad x \in X.$$

(U) X and Y are nonempty sets. f is upward [19] on Y, that is, $-f$ is downward on Y.

(U^*) X and Y are nonempty sets. f is weakly upward [4, 1, 2] on Y, that is, $-f$ is weakly downward on Y.

Under certain topological conditions on X, Y and f, any combination of convexity-concavity from (I) and (II) yields a minimax theorem.

In fact, the minimax theorems involving the conditions (C_*)–(C^*) were given by König in [12, 13], and by Ricceri in [17]; the minimax theorems involving the conditions (C_*)–(S^*) (or (S_*)–(C^*)) were given by Cheng-Lin in [3]; the minimax theorems involving the conditions (D_*)–(S^*) (or (S_*)–(U^*)) were given by Cheng-Lin in [2]; the minimax theorems involving the conditions (D)–(C^*) (or (C_*)–(U)) were given by Simons in [18]; the minimax theorems involving the conditions (D)–(U) were given by Simons in [19]; the minimax theorems involving the conditions (D_*)–(C^*) (or (C_*)–(U^*)) were given by Cheng-Lin-Yu in [1]; the minimax theorems involving the conditions

(D_*)–(U^*) were given by Cheng-Lin-Yu in [1], by Domokos in [4] and by Kindler in [10]; the minimax theorems involving the conditions (D_*)–(U) (or (D)–(U^*)) were given by Kindler [10].

The cases that have not been given are minimax theorems involving the following two sets of conditions:

(i) (D)–(S^*) (or (S_*)–(U));

(ii) (S_*)–(S^*).

In this paper, we shall give two minimax theorems (Theorems 2 and 3 in Section 3) under weaker conditions than the condition (i) or (ii), and give a slight generalization (Theorem 1 in Section 3) of Kindler's minimax theorem involving the condition (D_*)–(U). In our theorems, it is not required that the space X or Y is compact.

2. Preliminaries

Definition. Let X and Y be two nonempty set. Let $f : X \times Y \to \mathbf{R}$ be a function.

(FD_*) f is said to be *finitely weakly downward* [10] on X, if, for every x_1, x_2 in X and every finite subset A of Y, there exists x_0 in X such that for all $y \in A$, (A) holds, and for all $y \in \left\{ y \in A : f(x_1, y) \neq f(x_2, y) \right\}$, (C) holds.

(FD) f is said to be *finitely downward* on X, if, for every $\varepsilon > 0$, there exists $\delta > 0$ such that for every x_1, x_2 in X and every finite subset A of Y, there exists x_0 in X such that for all $y \in A$, (A) holds, and for all $y \in \left\{ y \in A : \left| f(x_1, y) - f(x_2, y) \right| \geqq \varepsilon \right\}$, (B) holds.

(CLD) f is called *compactly locally downward* on X, if, for every $\varepsilon > 0$, there exists $\delta > 0$ such that for every x_1, x_2 in X and every compact subset K of Y (where Y is required to be a topological space), there exists x_0 in X such that for all $y \in Y$, (A) holds, and for all $y \in \left\{ y \in K : \left| f(x_1, y) - f(x_2, y) \right| \geqq \varepsilon \right\}$, (B) holds.

It is easy to see that the following conclusion is true:

Lemma 1. *Let X and Y be two nonempty sets. Let $f : X \times Y \to \mathbf{R}$ be a function. Then, on X,*

$$\begin{gathered} f \text{ is downward} \\ \Downarrow \\ f \text{ is compactly locally downward} \\ \Downarrow \\ f \text{ is finitely downward} \\ \Downarrow \\ f \text{ is finitely weakly downward;} \end{gathered}$$

and

$$f \text{ is weakly downward} \Downarrow f \text{ is finitely weakly downward.}$$

Similarly, f may be defined as *finitely weakly upward, finitely upward,* or *compactly locally upward* on Y. The corresponding statement of Lemma 1 also holds.

For convenience, for any finite subset A of Y and any $r \in \mathbf{R}$, we denote

$$U^r(A) = \bigcap_{y \in A} \{ x \in X : f(x,y) \geqq r \}.$$

A family $\mathcal{H}$ of subsets of X is said to be pseudoconnected [20] if for any H_0, H_1, H_2 in $\mathcal{H}$, $H_0 \cap H_1 \neq \emptyset \neq H_0 \cap H_2$ and $H_0 \subset H_1 \cup H_2$ imply that $H_1 \cap H_2 \neq \emptyset$.

LEMMA 2. *Let X be a topological space and let Y be a nonempty set. Let $f : X \times Y \to \mathbf{R}$ be a function such that $U^r(y)$ is a compact subset of X for any $y \in Y$ and $r \in \mathbf{R}$. Suppose that one of the following conditions is satisfied:*

(FD_*) *f is finitely weakly downward on X;*

(S_*) *X is an interval space and for any $x_1, x_2 \in X$ and for all $x \in [x_1, x_2]$,*

$$f(x,y) \geqq \min \{ f(x_1,y), f(x_2,y) \} \quad \text{for all} \quad y \in Y. \tag{D}$$

Then for any finite subset A of Y and any r in $\mathbf{R}$, the family $\{U^r(y) \cap U^r(A)\}_{y \in Y}$ is pseudoconnected.

PROOF. Suppose that $A \subset Y$ is finite and $r \in \mathbf{R}$. Let y_0, y_1, y_2 in Y such that

$$U^r(y_0) \cap U^r(A) \subset U^r(y_1) \cup U^r(y_2),$$

$$U^r(y_0) \cap U^r(y_1) \cap U^r(A) \neq \emptyset, \quad U^r(y_0) \cap U^r(y_2) \cap U^r(A) \neq \emptyset$$

and

$$U^r(y_1) \cap U^r(y_2) \cap U^r(A) = \emptyset.$$

Let $D = U^r(y_0) \cap U^r(A)$ and $D_i = D \cap U^r(y_i)$, $i = 1, 2$. Then

$$D \subset D_1 \cup D_2, \quad D_1 \neq \emptyset \neq D_2 \tag{1}$$

and

$$D_1 \cap D_2 = \emptyset. \tag{2}$$

Since $U^r(y)$ is compact for any $y \in Y$ and $r \in \mathbf{R}$, D_1 and D_2 are compact subsets in X and $f(\cdot, y)$ is usc (upper semicontinuous) on X, there exists $x_i \in D_i$, $i = 1, 2$, such that

$$f(x_1, y_2) = \max_{x \in D_1} f(x, y_2) \quad \text{and} \quad f(x_2, y_1) = \max_{x \in D_2} f(x, y_1).$$

By (2), $x_1 \notin D_2$ and $x_2 \notin D_1$. Hence $f(x_1, y_1) \geqq r > f(x_2, y_1)$ and $f(x_2, y_2) \geqq r > f(x_1, y_2)$.

(a) Suppose that condition (FD_*) is satisfied. Let $F = \{y_0, y_1, y_2\} \cup A$. Then for x_1, x_2 in X, there exists x_0 in X such that for all $y \in F$, (A) holds, and for all $y \in \{y \in F : f(x_1, y) \neq f(x_2, y)\}$, (C) holds.

Since $x_i \in D_i$ $(i = 1, 2)$, by (A) and (1), $x_0 \in D \subset D_1 \cup D_2$. If $x_0 \in D_2$, then by (C),

$$f(x_0, y_1) > \min\left\{ f(x_1, y_1), f(x_2, y_1)\right\} = f(x_2, y_1).$$

This contradicts the maximality of $f(x_2, y_1)$ in D_2. Similarly, if $x_0 \in D_1$, we get a contradiction with the maximality of $f(x_1, y_2)$ in D_1. Therefore $U^r(y_1) \cap U^r(y_2) \cap U^r(A) \neq \emptyset$. This completes the proof that the family $\left\{U^r(y) \cap U^r(A)\right\}_{y \in Y}$ is pseudoconnected for all finite subsets A in Y and $r \in \mathbf{R}$.

(b) Suppose that the condition (S_*) is satisfied. Take $x_i \in D_i$, $i = 1, 2$. For any $x \in [x_1, x_2]$, we have (D). Hence $[x_1, x_2] \subset D \subset D_1 \cup D_2$. Since D_i, $i = 1, 2$, are closed and $[x_1, x_2]$ is connected, this contradicts (2). Hence $U^r(y_1) \cap U^r(y_2) \cap U^r(A) \neq \emptyset$ and the family $\left\{U^r(y) \cap U^r(A)\right\}_{y \in Y}$ is pseudoconnected for all finite $A \subset Y$ and all $r \in \mathbf{R}$. $\square$

REMARK. Under the condition (S_*), it is only required that $U^r(y)$ is closed for any $y \in Y$ and $r \in \mathbf{R}$ in the proof of Lemma 2.

3. Main results

THEOREM 1. *Let X be a topological space and Y be a nonempty set. Let $f : X \times Y \to \mathbf{R}$ be a function such that $U^r(y)$ is a compact subset of X for any $y \in Y$ and $r \in \mathbf{R}$. Suppose that*

(CLU) *f is compactly locally upward on Y, that is, $-f$ is compactly locally downward on Y;*

(FD_*) *f is finitely weakly downward on X.*

Then $(*)$ holds.

PROOF. Since $U^r(y)$ is compact for any $y \in Y$ and $r \in R$, it suffices to prove that for any finite subset Y_0 of Y,

$$\sup_X \inf_{Y_0} f(x,y) \geqq \inf_Y \sup_X f(x,y). \tag{3}$$

We verify (3) by induction on the cardinality of Y_0.

It is clear that (3) holds when $\operatorname{card} Y_0 = 1$. Suppose that (3) is true when $\operatorname{card} Y_0 \leqq n$. For Y_0 with $\operatorname{card} Y_0 = n+1$, let $Y_0 = \{y_1, y_2\} \cup A$ where $\operatorname{card} A = n-1$. For any $y \in Y$, we have that (3) holds for $\{y\} \cup A$. Hence

$$\inf_{y\in Y} \sup_X \inf_{\{y\}\cup A} f(x,y) \geqq \inf_Y \sup_X f(x,y).$$

Let $\varepsilon > 0$ and let $r = \inf_{y\in Y} \sup_X \inf_{\{y\}\cup A} f(x,y) - 2\varepsilon$. Then for any $y \in Y$, there exists $x \in U^{r+\varepsilon}\big(\{y\} \cup A\big) \subset U^{r+\varepsilon}(y) \cap U^r(A)$. Hence for all $y \in Y$,

$$r + \varepsilon \leqq \sup_{x\in U^r(A)} f(x,y) < +\infty. \tag{4}$$

Let $h(y) = \sup_{x\in U^r(A)} f(x,y)$, $y \in Y$. By (4), $r \leqq \inf_{y\in Y} h(y)$. Since f is compactly locally upward on Y, for $\varepsilon > 0$, there exists $\delta > 0$ with the properties in (CLU).

We claim that there exist z_1, z_2 in Y such that

$$U^r(z_i) \cap U^r(A) \subset U^r(y_i) \tag{5}$$

and for all $y \in Y$,

$$h(y) \leqq h(z_i) - \delta \Longrightarrow U^r(y) \cap U^r(A) \not\subset U^r(z_i). \tag{6}$$

In fact, if y_i satisfies (6), take $z_i = y_i$. For any $z^1 \in Y$ satisfying (5) there exists $z^2 \in Y$ such that $h(z^2) \leqq h(z^1) - \delta$ and $U^r(z^2) \cap U^r(A) \subset U^r(z^1)$. Then z^2 satisfies (5). Take $z_i = z^2$ if z^2 also satisfies (6). Continuing the process, we get, for all $n \in \mathbf{N}$

$$h(z^1) \geqq h(z^2) + \delta \geqq \cdots \geqq h(z^n) + n\delta.$$

Since $\inf_{y\in Y} h(y) \geqq r$, the process must stop at some n. Take $z_i = z^n$.

Let $K = \big[U^r(y_1) \cup U^r(y_2)\big] \cap U^r(A)$. For $z_1, z_2 \in Y$ and the compact subset $K \subset X$, choose $y_0 \in Y$ as in the condition (CLU), i.e.,

$$f(x, y_0) \leqq \max\big\{f(x,z_1), f(x,z_2)\big\} \qquad \forall x \in X$$

and

$$f(x, y_0) \leqq \max\left\{ f(x, z_1), f(x, z_2)\right\} - \delta$$
$$\forall x \in \left\{ x \in K : \left| f(x, z_1) - f(x, z_2)\right| \geqq \varepsilon\right\}.$$

Hence

$$U^r(y_0) \cap U^r(A) \subset \left[U^r(z_1) \cap U^r(A)\right] \cup \left[U^r(z_2) \cap U^r(A)\right]. \tag{7}$$

Next, we prove that

$$U^r(y_0) \cap U^r(z_1) \cap U^r(A) \neq \emptyset. \tag{8}$$

Take $x_0 \in U^{r+\varepsilon}(y_0) \cap U^r(A) \subset U^r(y_0) \cap U^r(A)$. If $f(x_0, z_1) \geqq r$ then $x_0 \in U^r(y_0) \cap U^r(z_1) \cap U^r(A)$. If $f(x_0, z_1) < r < r + \varepsilon \leqq f(x_0, y_0) \leqq f(x_0, z_2)$, this implies that

$$x_0 \in \left\{ x \in K : \left| f(x, z_1) - f(x, z_2)\right| \geqq \varepsilon\right\}.$$

By (CLU)

$$h(z_2) - \delta \geqq f(x_0, z_2) - \delta = \max\left\{ f(x_0, z_1), f(x_0, z_2)\right\} - \delta \geqq f(x_0, y_0).$$

Since

$$h(y_0) = \sup_{x \in U^r(A)} f(x, y_0) = \sup_{x \in U^r(A) \cap U^{r+\varepsilon}(y_0)} f(x, y_0),$$

it follows that

$$h(z_2) - \delta \geqq h(y_0).$$

By (6)

$$U^r(y_0) \cap U^r(A) \not\subset U^r(z_2).$$

Now (8) follows from (7).

Similarly, we can prove that

$$U^r(y_0) \cap U^r(z_2) \cap U^r(A) \neq \emptyset. \tag{9}$$

From (7),(8),(9) and Lemma 2, it follows that $U^r(z_1) \cap U^r(z_2) \cap U^r(A) \neq \emptyset$. Hence $U^r(y_1) \cap U^r(y_2) \cap U^r(A) \neq \emptyset$ by (5). Let $x \in U^r(y_1) \cap U^r(y_2) \cap U^r(A)$. Then $\inf\limits_{\{y_1, y_2\} \cup A} f(x, y) \geqq r$. Hence $\sup\limits_X \inf\limits_{Y_0} f(x, y) \geqq r$. Since $\varepsilon > 0$ is arbitrary, it follows that

$$\sup_X \inf_{Y_0} f(x, y) \geqq \inf_{y \in Y} \sup_X \inf_{\{y\} \cup A} f(x, y) \geqq \inf_Y \sup_X f(x, y). \qquad \square$$

COROLLARY 1 [10]. *Let X be a compact space and let Y be a nonempty set. Let $f : X \times Y \to \mathbf{R}$ be a function such that $f(\cdot, y)$ is usc on X for all $y \in Y$. Suppose that* (U) *and* (FD$_*$) *hold*

Then $(*)$ *holds.*

PROOF. Since the condition that $f(\cdot, y)$ is usc on the compact space X for any $y \in Y$ implies that $U^r(y)$ is compact for any $y \in Y$ and $r \in \mathbf{R}$, and (U) implies (CLU) by Lemma 1, Corollary 1 follows from Theorem 1. $\square$

Similarly, it is clear that the next Corollary 2 follows from Theorem 1.

COROLLARY 2 [19]. *Let X be a compact space and let Y be a nonempty set. Let $f : X \times Y \to \mathbf{R}$ be a function such that $f(\cdot, y)$ is usc on X for all $y \in Y$. Suppose that* (U) *and* (D) *hold.*

Then $(*)$ *holds.*

THEOREM 2. *Let X be a topological space and let Y be an interval space. Let $f : X \times Y \to \mathbf{R}$ be a function such that $U^r(y)$ is a compact subset of X for any $y \in Y$ and $r \in \mathbf{R}$, and $f(x, \cdot)$ is lsc (lower semicontinuous) on any interval of Y for any x in X. Suppose that* (S*) *and* (FD$_*$) *hold.*

Then $(*)$ *holds.*

PROOF. Since f is finitely weakly downward on X, by Lemma 2, it follows that the family $\{U^r(y) \cap U^r(A)\}_{y \in Y}$ is pseudoconnected for all finite subsets A of Y and all $r \in \mathbf{R}$. Theorem 2 follows by using the same argument as the proof of Theorem given in [2]. $\square$

COROLLARY 3. *Let X be a compact space and let Y be an interval space. Let $f : X \times Y \to \mathbf{R}$ be a function such that $f(\cdot, y)$ is usc on X for any y in Y and $f(x, \cdot)$ is lsc on any interval of Y for any x in X. Suppose that* (S*) *and* (FD) *hold.*

Then $(*)$ *holds.*

THEOREM 3. *Let X and Y be two interval spaces. Let $f : X \times Y \to \mathbf{R}$ be a function such that $U^r(y)$ is a closed subset of X for any $y \in Y$ and $r \in \mathbf{R}$, $U^{r_0}(y_0)$ is a compact subset of X for some $y_0 \in Y$ and $r_0 < \inf_Y \sup_X f(x, y)$, and $f(x, \cdot)$ is lsc on any interval of Y for all x in X. Suppose that* (S*) *and* (S$_*$) *hold.*

Then $(*)$ *holds.*

PROOF. By Lemma 2 and Remark, the condition (S$_*$) implies that the family $\{U^r(y) \cap U^r(A)\}_{y \in Y}$ is pseudoconnected for all finite subsets A of Y and for all $r \in \mathbf{R}$. Theorem 3 is proved as the proof of Theorem 2. $\square$

References

[1] Cao-Zong Cheng, Bor-Luh Lin and Feng-Shuo Yu, Weakly upward-downward minimax theorem, in: *Minimax Theory and Applications,* B. Ricceri and S. Simons, Eds, Kluwer Acad. Pub. (1998), pp. 21–28.

[2] Cao-Zong Cheng and Bor-Luh Lin, A minimax theorem involving weakly downward functions, *Acta Math. Hungar.,* **87** (2000), 287–293.

[3] Cao-Zong Cheng and Bor-Luh Lin, A two functions, noncompact topological minimax theorem, *Acta Math. Hungar.,* **73** (1996), 65–69.

[4] A. Domokos, A minimax theorem, *Annales Univ. Sci. Budapest,* **37** (1994), 157–163.

[5] Ky Fan, Minimax theorems, *Proc. Nat. Acad. Sci. U.S.A.,* **39** (1953), 42–47.

[6] M. A. Geraghty and Bor-Luh Lin, On a minimax theorem of Terkelsen, *Bull. Inst. Math. Acad. Sinica,* **11** (1983), 343–347.

[7] M. A. Geraghty and Bor-Luh Lin, Topological minimax theorems, *Proc. Amer. Math. Soc.,* **91** (1984), 337–380.

[8] M. A. Geraghty and Bor-Luh Lin, Minimax theorems without linear structure, *Linear and Multilinear Algebra,* **17** (1985), 171–180.

[9] M. A. Geraghty and Bor-Luh Lin, Minimax theorems without convexity, *Contemporary Math.,* **52** (1986), 102–108.

[10] J. Kindler, *Domokos' minimax theorem for usc-lsc functions,* (to appear).

[11] H. König, Über das von Neumannsche Minimax-Theorem, *Arch. Math.,* **19** (1968), 482–487.

[12] H. König, A general minimax theorem based on connectedness, *Arch. Math.,* **59** (1992), 55–64.

[13] H. König, Addendum to "A general minimax theorem based on connectedness", *Arch. Math.,* **64** (1995), 139–143.

[14] H. König and M. Neumann, *Mathematische Wirtschaftstheorie — mit einer Einführung in die konvexe Analysis,* Mathematical Systems in Economics 100, A. Hain 1986.

[15] Bor-Luh Lin and Xiu-Chi Quan, A symmetric minimax theorem without linear structure, *Arch. Math.,* **52** (1989), 367–370.

[16] M. Neumann, Bemerkungen zum von Neumannschen Minimaxtheorem, *Arch. Math.,* **29** (1977), 96–105.

[17] B. Ricceri, Some topological minimax theorems via an alternative principle for multifunctions, *Arch. Math.,* **60** (1993), 367–377.

[18] S. Simons, On Terkelsen's minimax theorem, *Bull. Inst. Math. Acad. Sinica,* **18** (1990), 35–39.

[19] S. Simons, An upward-downward minimax theorem, *Arch. Math.,* **55** (1990), 275–279.

[20] S. Simons, A flexible minimax theorem, *Acta Math. Hungar.,* **63** (1994), 119–132.

[21] S. Simons, Minimax theorems and their proofs, in: *Minimax Theory and Applications,* D.-Z. Du and P. M. Pardalos, Eds., Kluwer Acad. Pub. (1995), pp. 1–23.

[22] L. L. Stachó, Minimax theorems beyond topological vector spaces, *Acta Sci. Math. (Szeged)*, **42** (1980), 157–164.

[23] Wen-Tsun Wu, A remark on the fundamental theorem in the theory of games, *Sci. Rec. New Series,* **3** (1959), 229–233.

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