REPORT

Haifa 1985 Conference on Matrix Theory

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INTRODUCTION

The Haifa 1985 Conference is the second in a series (hopefully of length greater than 2) of conferences on matrix theory. It was held from 29–31 December, 1985 under the auspices of the Israel Mathematical Union, and hosted by the Department of Mathematics at the Technion and the Department of Mathematics and Computer Science at the University of Haifa. Participants from Israeli universities, research institutes, and high-tech industries gathered to listen to twenty-seven speakers, including three guests from the U.S.A. and West Germany. The informal exchange of information and ideas culminated in an open problem session.

The social program included receptions at the University of Haifa and at the Technion, and a guided tour of the Reuben and Edith Hecht Museum at the University of Haifa.

This report contains synopses of talks presented at the meeting, which were made available to us. They are arranged in alphabetical order. In coauthored synopses, the speakers' names are starred.

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The program was as follows:

29 December
Technion, The Silver Institute of Biomedical Engineering Auditorium

10:00–11:00

Dimension of faces generated by certain positive linear operators, by R. Loewy, Technion.
Sign patterns of matrices and their inverses, by M. Berger, Weizmann Institute.
Chairman: A. Berman, Technion.

11:30–12:30

Submultiplicativity and mixed submultiplicativity of matrix norms and operator norms, by M. Goldberg, Technion.
The resolvent condition and uniform power boundedness, by E. Tadmor, Tel-Aviv University.
Chairman: H. Wolkowicz, University of Delaware.

14:00–15:00

An efficient preconditioning algorithm and its analysis, by I. Efrat, IBM Scientific Center, Haifa.
Parallel algorithms for triangular systems, by A. Lin, Technion.
Chairman: I. Gohberg, Tel-Aviv University.

15:30–16:30

Minimality and irreducibility of time-invariant boundary-value systems, by L. Lerer, Technion.
Maximum-entropy extensions of matrices and related problems, by I. Gohberg, Tel-Aviv University.
Chairman: I. Cederbaum, Technion.

16:30–17:00

Open-problems session
Chairman: D. Hershkowitz, Technion.

30 December
University of Haifa, Main Building, Room 608

9:00–10:30

Class functions of finite groups, nonnegative matrices, and generalized circulants, by D. Chillag, Technion.

On linearly constrained entropy maximization, by Y. Censor, University of Haifa.
Some theorems in matrix theory using optimization, by H. Wolkowicz, University of Delaware.
Chairman: J. Sonn, Technion.

11:00–12:30

Holdability, irreducibility, and *M*-matrices, by A. Berman, Technion.
On the uniqueness of the Lyapunov scaling factors, by D. Shasha, Technion.
On positive reciprocal matrices, by V. Mehrmann, University of Bielefeld.
Chairman: H. Schneider, University of Wisconsin and Technion.

14:00–15:30

On biholomorphic automorphisms of the unit ball of unitary matrix spaces, by J. Arazy, University of Haifa.
Perturbed and mixed Toeplitz matrices as generalization of the resultant matrix, by B. Kon, Technion.
Bezoutian for several matrix polynomials and polynomial Lyapunov-type equations, by M. Tismenetsky, IBM Scientific Center, Haifa.
Chairman: G. Moran, University of Haifa.

16:00–17:00

Powers of a nonnegative definite matrix related to interpolation by radial functions, by N. Dyn, Tel-Aviv University.
Using Gauss-Jordan elimination to compute the index, null space, and Drazin inverse, by U. Rothblum, Technion.
Chairman: A. Pinkus, Technion.

31 December
Technion, The Silver Institute of Biomedical Engineering Auditorium

9:30–10:30

Higher dimensional Euclidean and hyperbolic matrix spaces, by B. Schwarz, Technion.
From the complex numbers to complex matrices along the projective line, by A. Zaks, Technion.
Chairman: D. London, Technion.

11:00–12:00

Maximum-distance problem and band sequences, by A. Ben-Artzi, Tel-Aviv University.

Block-Hankel-matrix inversion and the partial-realization problem, by P. A. Fuhrmann, Ben-Gurion University at the Negev.

Chairman: V. Mehrmann, University of Bielefeld.

13:30–14:30

Eigenstructures and signal processing, by A. Bruckstein, Technion.

The structure of root clustering criteria, by S. Gutman, Technion.

Chairman: P. A. Fuhrmann, Ben-Gurion University at the Negev.

15:00–16:00

Matrices with sign symmetric diagonal shifts, by D. Hershkowitz, Technion.

Equality classes of matrices: The extremal case of an inequality due to Ostrowski, by H. Schneider, University of Wisconsin and Technion.

Chairman: Y. Censor, University of Haifa.

Synopses of the talks are presented below.

COMPUTING THE INDEX AND DRAZIN INVERSE USING THE SHUFFLE ALGORITHM

by KURT M. ANSTREICHER[1] and URIEL G. ROTHBLUM[2]*

The well-known Gauss-Jordan elimination procedure computes the inverse of a nonsingular matrix A by executing elementary row operations on the pair (A, I) to transform it into (I, A^{-1}). Moreover, Gauss-Jordan elimination can be used to determine whether or not a matrix A is nonsingular, in the case where this fact is not known *a priori*. We adapt the Gauss-Jordan elimination procedure via "shuffles" to obtain an algorithm which computes the index of a given matrix A and determines bases of the null spaces of the powers of A. In the worst case the algorithm requires less than $2n^3$ arithmetic operations, compared to the well-known bound of n^3 operations for the work needed to invert a nonsingular matrix using Gauss-Jordan elimination. Moreover, the procedure suggests adaptation of efficient techniques for computing inverses

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(cf. [13]). Finally, the Drazin inverse of the underlying matrix A has a simple representation in terms of the output of the algorithm and the matrix A itself.

An algorithm incorporating shuffle operations was first devised by Luenberger [9], who applied it to the study of singular difference equations. Anstreicher [1] analyzed Luenberger's algorithm as applied to the solution of singular systems of linear differential equations with constant coefficients. Previous methods for solving the latter used Drazin inverses (e.g., [3] and [12]), suggesting a connection between the shuffle algorithm and the Drazin inverse. These methods are closely related to the study of matrix pencils (see [11] and references therein).

To introduce our new algorithm we consider an example (cf. [2, p. 132]). Let

$$A=\begin{pmatrix} 2 & 4 & 6 & 5\\ 1 & 4 & 5 & 4\\ 0 & -1 & -1 & 0\\ -1 & -2 & -3 & 3 \end{pmatrix}.$$

Elementary row operations transform (A, I) into

$$\begin{pmatrix} 1 & 0 & 1 & 0 & \frac{4}{3} & -\frac{5}{3} & -\frac{4}{3} & 0\\ 0 & 1 & 1 & 0 & 0 & 0 & -1 & 0\\ 0 & 0 & 0 & 1 & -\frac{1}{3} & \frac{2}{3} & \frac{4}{3} & 0\\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{2}{3} & 1 \end{pmatrix}.$$

It is clear that A is not invertible, as a linear combination of its rows vanishes. The "shuffle step" will next exchange row(s) of zeros with the corresponding row(s) of the right-hand matrix. This yields

$$\begin{pmatrix} 1 & 0 & 1 & 0 & \frac{4}{3} & -\frac{5}{3} & -\frac{4}{3} & 0\\ 0 & 1 & 1 & 0 & 0 & 0 & -1 & 0\\ 0 & 0 & 0 & 1 & -\frac{1}{3} & \frac{2}{3} & \frac{4}{3} & 0\\ \frac{1}{3} & \frac{1}{3} & \frac{2}{3} & 1 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

One then resumes elementary row operations, which result in

$$\begin{pmatrix} 1 & 0 & 1 & 0 & \frac{4}{3} & -\frac{5}{3} & -\frac{4}{3} & 0\\ 0 & 1 & 1 & 0 & 0 & 0 & -1 & 0\\ 0 & 0 & 0 & 1 & -\frac{1}{3} & \frac{2}{3} & \frac{4}{3} & 0\\ 0 & 0 & 0 & 0 & -\frac{1}{3} & -\frac{1}{3} & -\frac{5}{3} & 0 \end{pmatrix}.$$

A second shuffle is next performed, yielding

$$\begin{pmatrix} 1 & 0 & 1 & 0 & \frac{4}{3} & -\frac{5}{3} & -\frac{4}{3} & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{3} & \frac{2}{3} & \frac{4}{3} & 0 \\ -\frac{1}{3} & -\frac{1}{3} & -\frac{5}{3} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Elementary row operations are now finally used to convert the left-hand matrix to the identity, yielding

$$\begin{pmatrix} 1 & 0 & 0 & 0 & \frac{16}{9} & -\frac{20}{9} & -\frac{19}{9} & 0 \\ 0 & 1 & 0 & 0 & \frac{4}{9} & -\frac{5}{9} & -\frac{16}{9} & 0 \\ 0 & 0 & 1 & 0 & -\frac{4}{9} & \frac{5}{9} & \frac{7}{9} & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{3} & \frac{2}{3} & \frac{4}{3} & 0 \end{pmatrix}.$$

at which point the algorithm terminates.

Our main results are as follows. First, the algorithm always terminates in a finite number of shuffling steps, and this number equals the index of the underlying matrix A, say ν. In the above example, the number of shuffling steps is two, so $\nu = 2$. Second, the rows shuffled in the first through kth shuffle steps, $k = 1, 2, \ldots$, form a basis of the (row) null space $\{x : x^{\mathsf{T}}A^k = 0\}$. In the above example, $\{(\frac{1}{3}, \frac{1}{3}, \frac{2}{3}, 1)\}$ is a basis of $\{x : x^{\mathsf{T}}A = 0\}$, and $\{(\frac{1}{3}, \frac{1}{3}, \frac{2}{3}, 1), (-\frac{1}{3}, -\frac{1}{3}, -\frac{5}{3}, 0)\}$ is a basis of $\{x : x^{\mathsf{T}}A^2 = 0\}$. Finally, we show that A^D, the Drazin inverse of A, can be obtained from the matrix $\hat{A}$ on the right-hand side of the terminating matrix by computing $\hat{A}^{\nu+1}A^{\nu}$. In the above example

$$A^D = 9^{-3}\begin{pmatrix} 16 & -20 & -19 & 0 \\ 4 & -5 & -16 & 0 \\ -4 & 5 & 7 & 0 \\ -3 & 6 & 12 & 0 \end{pmatrix}^3 \begin{pmatrix} 2 & 4 & 6 & 5 \\ 1 & 4 & 5 & 4 \\ 0 & -1 & -1 & 0 \\ -1 & -2 & -3 & 3 \end{pmatrix}^2$$

$$= \begin{pmatrix} 3 & -1 & 2 & 2 \\ 2 & 1 & 3 & 3 \\ -1 & 0 & -1 & -1 \\ -1 & 0 & -1 & -1 \end{pmatrix}.$$

We next describe our algorithm in detail. Consider a given $n \times n$ real matrix A. In the course of the algorithm a sequence of pairs of matrices

$(A^{(k)}, B^{(k)})$, is generated, where $(A^{(0)}, B^{(0)}) = (A, I)$. Given $(A^{(k)}, B^{(k)})$ we execute row operations on $A^{(k)}$ to convert it into a matrix whose nonzero rows are linearly independent; moreover, if $A^{(k)}$ is found to be nonsingular the algorithm terminates. Simultaneously, we execute the same row operations on $B^{(k)}$. Let $\overline{A}^{(k)}$ and $\overline{B}^{(k)}$ be the result of executing the above row operations on $A^{(k)}$ and $B^{(k)}$, respectively. If $\overline{A}^{(k)}$ has zero rows, we exchange these rows with the corresponding rows of $\overline{B}^{(k)}$ and proceed to iteration $k+1$. We show that if ν is the index of A, then the algorithm will always terminate on exactly the νth iteration. Moreover, the rows shuffled on iterations $0, \ldots, k-1$, for $k = 1, \ldots, \nu$, are a basis of the left null space of A^k. In addition, we show that if on iteration ν, $A^{(\nu)}$ is transformed into the identity matrix, $\overline{A}^{(\nu)} = I$, and $\hat{A}$ is defined to be the resulting matrix $\overline{B}^{(\nu)}$, then the Drazin inverse of A is equal to $\hat{A}^{\nu+1}A^{\nu}$.

A representation of the Drazin inverse of matrices for which zero is a simple eigenvalue is given in [8, Lemma 5.1]. The representation in this case (for which the index is known to be one) reduces to the execution of our shuffle algorithm. A special case of the above representation for matrices having the form *I-P*, where *P* is an irreducible stochastic matrix, is given in [4, Theorem 8].

A survey of methods for computing the index and Drazin inverse of a matrix can be found in [2]. In particular, efficient methods for computing the Drazin inverse are given in [5] and [6, 7].

Further details and proofs concerning the new shuffle algorithm can be found in a forthcoming paper of the authors that will appear in this journal.

The authors wish to thank Eric V. Denardo for illuminating comments concerning the operation of the algorithm, and constructive criticism of the paper.

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ON BIHOLOMORPHIC AUTOMORPHISMS OF THE UNIT BALL OF UNITARY MATRIX SPACES

by JONATHAN ARAZY[3]

Let S denote the space of all complex matrices $a=(a_{i,j})_{i,j=1}^{\infty}$ with only finitely many nonzero entries. A norm α on S is *unitarily invariant* if

$$\alpha(uaw)=\alpha(a)$$

for all $a \in S$ and all unitary matrices u, w with the property that $u_{i,j}=w_{i,j}=\delta_{i,j}$ if $\max\{i,j\}$ is large enough. For normalization one requires also that $\alpha(a)=1$ for every rank-one partial isometry a in S.

The unitary matrix space S_α associated with α is the completion of S under α (these spaces are called also "unitary ideals" or "symmetric normed ideals"; see [1]). We denote by B_α the open unit ball of S_α. It is known that the most general unitarily invariant norm α on S is given by

$$\alpha(a)=\beta(s_n(a)),$$

where $s_n(a)=\lambda_n((a^*a)^{1/2})$, $n=1,2,\dots$, are the singular numbers of a, and

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β is a symmetric norm on sequences. For $1 \leqslant p < \infty$ we put

$$\alpha_p(a)=\left(\sum_{n=1}^{\infty} s_n(a)^p\right)^{1/p}=\|(s_n(a))\|_{l_p}$$

and

$$\alpha_\infty(a)=s_1(a)=\text{the operator norm of } a.$$

We let $S_p=S_{\alpha_p}$. Thus S_1 is the trace class, S_2 is the space of Hilbert-Schmidt operators, and S_∞ is the space of compact operators on l_2.

It is well known that B_2 and B_∞ are bounded symmetric domains, that is the corresponding groups of biholomorphic automorphisms act transitively (in fact, by Möbius transformations; see [2] and [5]). If these two cases are ruled out, the situation changes drastically.

THEOREM 1. *Let α be a unitarily invariant norm, different from α_2 and α_∞. Then every biholomorphic automorphism of B_α extends to a linear isometry of S_α.*

With the aid of [3] we conclude

COROLLARY. *Let α be as in Theorem 1, and let F be a biholomorphic automorphism of B_α. Then there exist unitary matrices u, w so that either*

$$F(a)=uaw, \qquad a\in S_\alpha,$$

or

$$F(a)=ua^Tw, \qquad a\in S_\alpha,$$

where a^T is the transpose of a.

A holomorphic vector field $X: B_\alpha \to S_\alpha$ is called *complete* if there exists a solution $\phi=\phi_X: R\times B_\alpha \to B_\alpha$ to the initial-value problem

$$\frac{\partial}{\partial t}\phi(t,a)=X(\phi(t,a)),$$

$$\phi(0,a)=a$$

for every $a\in B_\alpha$.

THEOREM 2. *Let α be a unitarily invariant norm, different from α_2 and α_∞. Then every complete holomorphic vector field $X: B_\alpha \to S_\alpha$ extends to a bounded, linear, skew-Hermitian operator on S_α.*

Here "skew-Hermitian" means that the numerical range is purely imaginary. Again, using [3] we get

COROLLARY. *Let α and $X: B_\alpha \to S_\alpha$ be as in Theorem 2. Then there exist bounded Hermitian matrices b, c so that*

$$X(a) = i(ba + ac), \qquad a \in S_\alpha.$$

The proofs of Theorems 1 and 2 use the "contraction principle" of [4].

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THE MAXIMUM DISTANCE PROBLEM AND BAND SEQUENCES

by A. BEN-ARTZI,[4]* R. L. ELLIS,[5] I. GOHBERG,[4] and D. LAY[5]

In this paper we solve the following problem. For $1 \leqslant j, k \leqslant n$ and $|j-k| \leqslant m$, let a_{jk} be a given complex number with $a_{kj} = \bar{a}_{jk}$. We wish to find linearly independent vectors $x_1, \dots, x_n$ such that $\langle x_k, x_j \rangle = a_{jk}$ for $|j-k| \leqslant m$ and such that the distance from x_k to the linear span of $x_1, \dots, x_{k-1}$ is maximal for $2 \leqslant k \leqslant n$. We construct and characterize all such sequences of vectors. Our solution leads naturally to the class of m-band sequences of vectors in an inner-product space. We study these sequences and characterize their equivalence classes under unitary transformations. A precise formulation of the problem now follows.

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Maximum-Distance Problem

For any vectors $x_1, \dots, x_n$ in C^n and for $2 \leqslant k \leqslant n$, let $\mathrm{sp}\langle x_1, \dots, x_k \rangle$ denote the subspace spanned by $x_1, \dots, x_k$, and let $\mathrm{dist}(x_k, \mathrm{sp}\langle x_1, \dots, x_{k-1}\rangle)$ denote the distance from x_k to $\mathrm{sp}\langle x_1, \dots, x_{k-1}\rangle$. Given $0 \leqslant m < n$ and a set $\{a_{jk} : |j-k| \leqslant m\}$ of complex numbers satisfying $a_{kj} = \bar{a}_{jk}$, we shall say that a sequence of vectors $\{x_k\}_{k=1}^n$ is admissible if it is linearly independent and if

$$\langle x_k, x_j \rangle = a_{jk} \qquad (|j-k| \leqslant m). \tag{1.1}$$

We define

$$d_k = \sup \mathrm{dist}(x_k, \mathrm{sp}\langle x_1, \dots, x_{k-1}\rangle) \qquad (2 \leqslant k \leqslant n),$$

where the supremum is taken over all admissible sequences of vectors in C^n. The maximum distance problem is to describe all admissible sequences $\{x_k\}_{k=1}^n$ such that

$$\mathrm{dist}(x_k, \mathrm{sp}\langle x_1, \dots, x_{k-1}\rangle) = d_k \qquad (2 \leqslant k \leqslant n).$$

Each such sequence is called a solution of the maximum-distance problem. This maximum-distance problem has close connections with maximum entropy in the mathematical theory of signal processing [2, 4, 5].

Band Sequences

The study of the maximum-distance problem leads naturally to the notion of a band sequence of vectors in C^n. If we apply the Gram-Schmidt process to a sequence of vectors $\{y_k\}_{k=1}^n$, we obtain a sequence $\{\omega_k\}_{k=1}^n$ of orthonormal vectors that is related to $\{y_k\}$ by a system of equations that may be written in the form

$$a_{1k}y_1 + \cdots + a_{kk}y_k = \omega_k \qquad (k = 1, \dots, n). \tag{1.2}$$

DEFINITION. Let m and n be integers with $0 \leqslant m < n$. We say that a linearly independent sequence $\{y_k\}_{k=1}^n$ is an m-band sequence if $a_{jk} = 0$ for $m < k \leqslant n$ and $1 \leqslant j < k - m$.