Inverse problem for 2-dimensional discrete Schrödinger operator

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We consider the discrete version the time independent Schrödinger equation

$$-\Delta u + qu = 0$$

with Dirichlet or Neumann boundary condition on Ω, where Ω is a bounded simply connected domain in $\mathbb{R}^n$. Here u is an element of a suitable function space, q is a potential function. The continuous inverse problem is the following: if the equation above is solvable for all boundary conditions, then with the knowledge of the Dirichlet-to-Neumann operator (which maps the Dirichlet boundary to the Neumann boundary conditions), is the potential q unique? This question came from the inverse conductivity problem, which was asked by Calderón [1]. The solution of this problem is for example in the article [5]: there is a natural connection between the continuous conductivity and Schrödinger equation.

Nowadays the research of discretized inverse problems is a hot topic. The discrete inverse conductivity problem has a powerful theory [2], but in the discrete case there is no longer connection between the conductivity and the Schrödinger problem. In this talk we use finite weighted graphs with boundary on Ω for the discretisation of the above problem.

After talking about the fundamental properties of the discrete Dirichlet-to-Neumann operator we investigate the question of unicity. There are some previous results for quadrilateral lattice, for example in two dimensions [4], in $d \geq 2$ dimensions for all energies [3], and I proved unicity for convex domains in $\mathbb{R}^2$.

Concentrating to the 2-dimensional case we consider the unit disc for Ω, which is conformal equivalent to all nonempty simply connected domain in $\mathbb{R}^2$. The discrete model on the disc is a circular planar graph G as in [2], which is critical in some special cases. We give a reconstruction algorithm for q in the interior nodes of G, thereby we prove unicity theorem for some special circular planar graphs.

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