Some geometric properties of neighbourly polytopes

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In this talk we are concerned with neighbourly polytopes in even dimensional spaces. Let $k \geq 2$ be an integer and let $d = 2k$. A d-dimensional polytope is neighbourly if any k-element subset of its vertex set determines a proper face F of P such that the vertex set of F coincides with the selected subset. In particular, if $d = 4$, then a 4-dimensional polytope is neighbourly precisely when any pair of its vertices forms an edge. The most well-known examples of neighbourly polytopes are the cyclic polytopes. Cyclic polytopes form a infinite class of neighbourly polytopes in all dimensions, and they are constructed as the convex hull of distinct points on the moment curve.

Neighbourly polytopes are important in geometry because among polytopes with a given number of vertices they have the maximum number of facets according to the upper bound theorem of McMullen [2]. There are only a few constructions that provide infinite families of such polytopes and even those constructions provide only a small fraction of all neighbourly polytopes. One of the most important construction techniques is the so-called *sewing* procedure of Shemer [1].

In this talk, we investigate the combinatorial and geometric properties of neighbourly 4-polytopes with few vertices that cannot be produced by sewing of Shemer [1].

This work is part of my Master's thesis written under the supervision of Ferenc Fodor (University of Szeged).

[1] I. Shemer, Neighborly polytopes, *Israel J. Math.* **43** (1982), 291–314.

[2] P. McMullen, The maximum numbers of faces of a convex polytope, *Mathematika* **17** (1970), 179–184.