A generalization of the Kaloujnine–Krasner Theorem

Tamás Dékány
University of Szeged, Szeged, Hungary

In this talk we will generalize a well-known theorem in group theory for some class of semigroups. The extensions of groups play a fundamental role both in the structure theory and in the theory of varieties of groups. In 1950, Kaloujnine and Krasner proved that for any groups N and H, every extension of N by H is embeddable in the wreath product of N by H, i.e. in a semidirect product of a well-chosen direct power of N by H.

An important class of the semigroups is the class of completely simple semigroups. These semigroups are disjoint unions of isomorphic groups, so they often appear in semigroup theory as generalizations of groups. In the present paper we investigate how to generalize the Kaloujnine–Krasner Theorem for the class of completely simple semigroups.

Any group congruence ρ of a completely simple semigroup S determines a normal subgroup N in every maximal subgroup G of S, and their union is the identity ρ-class, which is a completely simple subsemigroup in S, and S/ρ is isomorphic to G/N. On the one hand, we give a completely simple semigroup which is an extension of a completely simple semigroup K by a group H, and which is not embeddable in the wreath product of K by H. On the other hand we show that any extension of a completely simple semigroup K by a group H is embeddable in a semidirect product of a completely simple semigroup T by H, where the maximal subgroups of T are direct powers of the maximal subgroups of K.