On the solution of interval linear equation systems

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We describe two methods to bound the solution set of overdetermined full rank interval linear equation systems. In order to define the problem, let us denote the set of real compact intervals by $\mathbb{IR}$, the matrices and vectors whose elements are intervals by $\mathbf{A} \in \mathbb{IR}^{m\times n}$ and $\mathbf{b} \in \mathbb{IR}^m$ respectively. The purpose of this work is to give an enclosure interval vector of the solution set of the overdetermined systems of interval linear equations, $\mathbf{A}x = \mathbf{b}$. We suppose that $m \geq n$ and the interval matrix $\mathbf{A}$ has full rank, i.e., all real matrices $A \in \mathbf{A}$ have full rank. In recent years, much attention has been paid to systems of interval linear equations with square interval matrices (see, e.g., [1], [2], [3]). We note that it is especially interest if the matrix of the interval linear equation system is non-squared.

As we have seen in the square case, when solving the system $\mathbf{A}x = \mathbf{b}$ of linear equations using interval version of methods such as Gaussian elimination, it is generally advisable to precondition the system. The most commonly used method of preconditioning is to multiply the system by an approximate inverse of the midpoint matrix of $\mathbf{A}$. This preconditioning was introduced by E.R. Hansen in [4].

In our case, when the matrix of the interval linear equation is not square, there are two ways for preconditioning. In our first method we use the generalized inverse of the midpoint matrix of $\mathbf{A}$ to precondition the system. Our second method comes from the idea of the point (non-interval) case. Namely, we multiply the system by the transpose of the midpoint matrix of $\mathbf{A}$.

The proposed preconditioning results an $n\times n$ interval linear equation system. Then we bound the solution set of the preconditioned system applying the method provided by J. Rohn in [5]. Furthermore Gaussian elimination also can be used to bound the solution set of the preconditioned system. Finally we give some examples in which we compare the efficiency of our methods and compare the results with the interval Householder method [6].

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