Projective embeddings of 3- and 4-nets in perspective position

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A finite k-net of order n is an incidence structure consisting of $k \geq 3$ pairwise disjoint classes of lines, each of size n, such that every point incident with two lines from distinct classes is incident with exactly one line from each of the three classes. Deleting a line class from a k-net, with $k \geq 4$, gives a derived $(k-1)$-net of the same order. Finite k-nets embedded in a projective plane $PG(2, \mathbb{K})$ coordinatized by a field $\mathbb{K}$ of characteristic 0 only exist for $k = 3, 4$ as Korchmáros, Nagy and Pace showed in [1]. In this talk, we investigate 3-nets embedded in $PG(2, \mathbb{K})$ whose line classes are in perspective position with an axis r, that is, every point on the line r incident with two lines from different classes is incident with exactly one line from each class. The problem of determining all such 3-nets remains open whereas we obtain a complete classification for those coordinatizable by a group. As a corollary, for $n \neq 8$, the (unique) 4-net of order 3 embedded in $PG(2, \mathbb{K})$ is the only 4-net embedded in $PG(2, \mathbb{K})$ which has a derived 3-net coordinatized by a group G. This result remains valid in positive characteristic under the hypothesis that the order n of the k-net considered is smaller then the characteristic of $\mathbb{K}$, apart from possible sporadic cases occurring for $n \in \{12, 24, 60\}$ and $G \cong A_4, S_4, A_5$ respectively.

[1] G. Korchmáros, G. P. Nagy and N. Pace, k-nets embedded in a projective plane over a field, to appear in *Combinatorica* (2013).