Asymptotics for the estimation of the offspring means in critical two-type GWI processes

Kristóf Körmendi, Gyula Pap

University of Szeged, Szeged, Hungary

The model is as follows. Let $\boldsymbol{X}_k, k \in \mathbb{Z}_+$ be a two-dimensional stochastic process satisfying

$$\boldsymbol{X}_k = \sum_{j=1}^{X_{k-1,1}} \boldsymbol{\xi}_{k,j,1} + \sum_{j=1}^{X_{k-1,2}} \boldsymbol{\xi}_{k,j,2} + \boldsymbol{\varepsilon}_k \qquad k \in \mathbb{N},$$

where the nonnegative integer valued random vectors $\{\boldsymbol{\xi}_{k,j,i} : k, j \in \mathbb{Z}_+, i \in \{1, 2\}\}$ denote the number of offsprings of the jth individual who is type i in the $(k-1)^{\text{th}}$ generation, and $\{\boldsymbol{\varepsilon}_k : k \in \mathbb{Z}_+\}$ is the number of immigrants in the kth generation. Our aim is to estimate the offspring mean matrix

$$m_\xi := \begin{bmatrix} \mathbb{E}(\boldsymbol{\xi}_{1,1,1}) & \mathbb{E}(\boldsymbol{\xi}_{1,1,2}) \end{bmatrix},$$

in the critical case, that is when the spectral radius of this matrix is 1. We use the conditional least squares method to find an estimator and examine its asymptotic properties.

[1] M. Ispány, K. Körmendi and G. Pap, Asymptotic behavior of CLS estimators for 2-type doubly symmetric critical Galton–Watson processes with immigration, to appear in *Bernoulli*. Available on the ArXiv: `http://arxiv.org/abs/1210.8315`.