Online hypergraph coloring with and without rejection

Judit Nagy-György
University of Szeged, Szeged, Hungary

Let H be a hypergraph with vertex set $V(H)$ and edge set $E(H)$. A coloring is a *proper coloring* of H if no edge of H is monochromatic. The (proper) chromatic number of H is denoted by $\chi(H)$. A proper coloring of H is *conflict-free* if for each edge $e \in E(H)$, some color occurs on exactly one vertex of e. The conflict-free chromatic number of H is denoted by $\chi_{CF}(H)$. A *rainbow coloring* of H is a proper coloring of H such that for every edge $e \in E(H)$, the colors of all vertices of e are distinct. The rainbow chromatic number of H is denoted by $\chi_R(H)$. Rainbow coloring is conflict-free.

Online coloring without rejection. An online hypergraph coloring algorithm colors the i-th vertex of the hypergraph by only looking at the subhypergraph $H_i = (V_i, E_i)$ where V_i contains the first i vertices and E_i contains the edges of the hypergraph which are subsets of V_i. The cost of the algorithm is the number of the used colors.

Theorem 1 (Imreh, Nagy-György, [1])**.** *Let $k \geq 3$. For every online hypergraph coloring algorithm A there exists a 2-colorable k-uniform hypergraph H on n vertices with $\chi_A(H) \geq \lceil n/(k-1) \rceil$. If H is a k-uniform hypergraph then then First Fit algorithm uses at most $\lceil n/(k-1) \rceil$ colors.*

Theorem 2. *No online algorithm uses less then $n-1$ colors on 2-cf-colorable hypergraphs on n vertices. If H is a 2-cf-colorable hypergraph then First Fit algorithm uses at most $n-1$ colors if $n > 2$.*

Proposition 3. *No online algorithm uses less then n colors to rainbow-color hypergraphs on n vertices.*

Online coloring with rejection. If we allow *rejection* then all of vertices have penalty. In here the edges having rejected vertex are also rejected. The penalties of rejected vertices are added to the cost.

Theorem 4. *If $k \geq 3$, for every ε there is an online algorithm $\mathcal{A}_\varepsilon$ and n_ε, such that $\mathcal{A}_\varepsilon$ at most $\lceil n/(k-1) \rceil/2 + \varepsilon$ competitive on k-uniform 2-proper-colorable hypergraphs with at least n_ε vertices.*

Theorem 5. *There is an online algorithm with competitive ratio at most $(n-1)/\varphi + 1$ on 2-cf-colorable hypergraphs on n vertices where $\varphi = (1+\sqrt{5})/2$. No online algorithm exists which is $Cn+D$-competitive in the problem of conflict free coloring with rejection for hypergraphs containing n vertices and some constants $c < 1/\varphi$ and D.*

Proposition 6. *There is an online algorithm for rainbow coloring with rejection with competitive ratio n and there is no online algorithm with competitive ratio less then n.*

This is a joint work with Csanád Imreh.

This research was supported by the European Union and the State of Hungary, co-financed by the European Social Fund in the framework of TÁMOP-4.2.4.A/ 2-11/1-2012-0001 'National Excellence Program'.

[1] Cs. Imreh, J. Nagy-György, Online hypergraph coloring, *Information Processing Letters* **109** (2008), 23–26.