General discretization theory for nonlinear semigroups

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In paper [3] Sanz-Serna and Palencia proved a general Lax–Richtmyer–Kantorovich theorem for linear operator equations. The result based on the framework of Stetter and it can be applied to initial and boundary value problems discretized by finite elements, finite differences, etc. However, the abstract approach has stuck in. In the recent years we have made a similar approach to the investigation of the numerical solution of nonlinear operator equations in abstract settings. This work has been summarized in [1]. We also investigated the advantages of the introduced N- and K-stability notions. These result will be found in [2].

Sanz-Serna and Palencia mentioned in [3] that the well-known Lax theorem is a special case of their theorem for the linear abstract Cauchy problem. However, for the nonlinear abstract Cauchy problem only one convergence theorem exists ([4]). In this talk I would like to present how our abstract approach can be applied for the nonlinear abstract Cauchy problem with the help of the N-stability notion. It means that we will consider the following problem for an $A \in \mathcal{A}(\omega)$ accretive operator in the Banach space $\mathcal{X}$,

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t} u(t) &= A(u(t)), && t > 0, \\ u(0) &= u_0, && x \in \mathbb{R}, \end{aligned}$$

where $u_0 \in D(A)$. If the above problem has a solution for all $u_0 \in D(A)$ we define the solution $u(t) = S(t)u_0$, where $S(t)$ is a continuous nonlinear operator on $D(A)$ which can be then extended to $D := \overline{D(A)}$, where the family $\big(S(t)\big)_{t>0}$ of operators $S(t) : D \to D$ satisfies the usual strongly continuous semigroup identities. Then it is called a nonlinear semigroup of type ω.

For the above problem we will see that due to our approach not just the known, but other convergence theorems can be proved.

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[2] I. Fekete, I. Faragó, Stability concepts and their applications, *Comput. Math. Appl.* (2014), in press.

[3] J. M. Sanz-Serna, C. Palencia, A general equivalence theorem in the theory of discretization methods, *Math. Comp.* **45** (1985), no. 171, 143–152.

[4] A. Belleni-Morante, A. C. McBride, *Applied nonlinear semigroups. An introduction.*, Wiley Series in Mathematical Methods in Practice, 3. John Wiley & Sons, Ltd., Chichester, 1998.