

# A DIFFERENTIAL EQUATION MODEL OF OPTIMIZATION OF DATA TRANSFER RATE

**István Balázs**, Tibor Krisztin  
University of Szeged, Szeged, Hungary

We consider a system of differential equations that has a delay depending on the solution. The time delay is defined by an ordinary differential equation with a non-continuous right hand side. The problem emerges in optimization (in the function of utility and price) of data transfer rate of computer networks.

The equation can not be inserted neither in the standard theory of functional differential equations nor in the theory of equations with state-dependent delay emerging in the recent years. Two main technical problems cause the difficulty: the state-dependent delay and the not smooth member in the algebraic equation defining the delay.

The main result is that the system defines a continuous semi-dynamical system. Under certain conditions we verify global convergence to the optimum (which is an equilibrium). We also prove results for existence of periodic solutions around the optimum.

New results: construction of appropriate phase space for the problem, verifying existence and uniqueness of solution in the phase space and continuous dependence on initial data, furthermore showing possibility of periodic behavior.

- [1] F. KELLY, A. MAULLOO, D. TAN, Rate Control for Communication Networks: Shadow Prices, Proportional Fairness and Stability, *Journal of the Operational Research Society* **49(3)** (1998), 237–252.
- [2] P. RANJAN, R. J. LA, E. H. ABED, Global Stability with a State-Dependent Delay in Rate Control, *Proc. Conference on Time-Delay Systems, Belgium* (2004).