A computer-aided study of the delay model of Cheyne–Stokes respiration

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In our talk we deal with a delay differential equation model of the Cheyne–Stokes respiratory ailment. We assume that the removal of CO_2 from the blood is proportional to the product of the ventilation and the level of CO_2 in the blood. Let p be the constant metabolic production rate of CO_2 in the body. The dynamics of the CO_2 level is can be modelled by the equation

$$\frac{dc(t)}{dt} = p - bVc(t) = p - bV_{\max} c(t) \frac{c^m(t-\tau)}{a^m + c^m(t-\tau)}, \tag{1}$$

where $b > 0$ is a positive parameter which is also determined from experimental data. The delay time τ is the time between the oxygenation of the blood in the lungs and monitoring by the chemoreceptors in the brainstem. The justification for the equation is heuristic: the equation reproduces certain qualitative features of both normal and abnormal breathing.

In the talk, we present the theoretical as well as computer-aided investigations of the model. By applying the theory of linearization to the system we find conditions for the asymptotic stability of the equilibrium. Our dynamic demonstrations developed in Wolfram Mathematica allow us to study the dependence of the stability on the delay and the other physiologically significant parameters as well as the bifurcation values of them. Our experiments help to understand and investigate even finer properties and even the global dynamics of the solutions. Our developments can be used for dynamic study of other delay systems, and they can serve as teaching materials for courses on dynamic modelling.

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