Numerical methods for superdiffusion

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Anomalous diffusion processes play an important role in physics [5], chemistry [4] and biology [3]. Superdiffusion is a promising way for modeling turbulence [2], [1]. While classical diffusion is characterized in discrete models or observations by a linear time dependence of a second moment, anomalous diffusion shows a different time and spatial behavior. For these processes the second moment of the distribution scales as

$$\langle r^2(t)\rangle \propto t^{\frac{2}{\alpha}}, \text{ where } 0 < \alpha < 2.$$

To model superdiffusion we have to study the following initial-boundary value problem in two dimension:

$$\begin{cases} \frac{\partial u}{\partial t} = -\mu\,(-\Delta)^{\alpha}\,u, & \text{if } \underline{x} \in \Omega \\ u(\underline{x},0) = u_0(\underline{x}), & \text{if } \underline{x} \in \Omega, \\ u\Big|_{\partial\Omega} = g. \end{cases} \tag{1}$$

We have constructed a numerical solution of (1) using an implicit Euler time step and shown that the method is consistent and exponentially stable in the discrete $l_{2,h}$-norm. This is an important result for further work on modeling turbulence.

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