Investigation of the dynamics of the inverted pendulum using finite elements

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The motion of the inverted pendulum is modelled by the nonlinear second order differential equation

$$\ddot{x} - \frac{1}{\ell}\left(g + a\left(t\right)\right)\sin x = 0$$

where x means the angle between the vertical direction and the rod of the pendulum measured counterclockwise, g is the gravitational acceleration, ℓ is the length of the rod, and $a(t)$ is a T-periodic, piecewise constant function.

The finite element method is presented through the linearized equation. In this case, it is possible to compare the numerical solution with the analitical one. It is known that the upper equilibrium can be stabilized by appropriate oscillations of the suspension point [1].

Moreover, we examine an interesting application of the linear case, the Mathieu equation, where the coefficient of the linear part is a T-periodic, not piecewise constant function:

$$\ddot{x} - \cos\left(\frac{2\pi}{T}t\right)x = 0.$$

The numerical solution is compared with the finite element solution of the linear differential equation with the piecewise constant coefficient.

After that, we translate our method to the nonlinear equation. The nonlinearity is approximated with Taylor polynomials and the numerical solution of the consequent nonlinear differential equation is studied by the finite element method. As we do not know the exact solution we can only take observations what happens to the numerical solution when the degree of the Taylor polynomial is increased. The Poincaré map is also plotted.

Finally, we show simulations resulted from the applied method. In this research, we used the MATLAB programming language for implementation, computation and demonstration.

[1] L. Csizmadia, L. Hatvani An extension of the Levi–Weckesser method to the stabilization of the inverted pendulum under gravity, *Meccanica* **49** (2014), 1091–1100.