

$$(4) \sum_{n=3}^{\infty} \frac{(2n-1)^n}{2^{n-1} \cdot n^n} (x+1)^{2n+1}$$

$$\sum_{n=3}^{\infty} \frac{(2n-1)^n}{2^{n-1}} (x+1)^{2n+1} = \sum_{n=3}^{\infty} \frac{(2n-1)^n}{2^n \cdot \frac{1}{2}} (x+1)^{2n} \cdot (x+1) =$$

$$= \sum_{n=3}^{\infty} \frac{(2n-1)^n \cdot ((x+1)^2)^n \cdot (x+1)}{2^n \cdot \frac{1}{2}}$$

$$\lim_{x \rightarrow \infty} \int_3^{\infty} \sqrt[n]{\frac{(2x-1)^n \cdot ((x+1)^2)^n \cdot (x+1)}{2^n \cdot \frac{1}{2}}} dx = \lim_{x \rightarrow \infty} \int_3^{\infty} \frac{(2x-1) \cdot (x+1)^2 \cdot \sqrt[n]{x+1}}{2 \cdot \frac{1}{\sqrt{2}}} dx$$

$$= \lim_{x \rightarrow \infty} \int_3^{\infty} \frac{(2x^3 + 3x^2 - 1) \sqrt[n]{x+1}}{2 \cdot \frac{1}{\sqrt{2}}} dx = \frac{\sqrt[n]{x+1} (2x^3 + 3x^2 - 1)}{\sqrt[n]{x+1} (2x^3 + 3x^2 - 1)} dx =$$

$$\begin{aligned} (x+1)^2 &= x^2 + 1 + 2x \\ (x+1)^2 \cdot (2x-1) &= 2x^3 + 2x + 4x^2 - x^2 - 1 - 2x = 2x^3 + 3x^2 - 1 \end{aligned}$$

$$= \lim_{x \rightarrow \infty} \int_3^{\infty} \frac{(2x^3 + 3x^2 - 1)^{\frac{1}{n}} \cdot (x+1)}{\frac{2}{\sqrt{2}} \sqrt[n]{x+1} (2x^3 + 3x^2 - 1)} dx =$$

$$\lim_{x \rightarrow \infty} \int_3^{\infty} \sqrt[n]{\frac{(2x-1)^n \cdot (x+1)^{2n+1}}{2^{n-1}}} dx = \lim_{x \rightarrow \infty} \int_3^{\infty} \frac{(2x-1) \cdot (x+1)^{\frac{2n+1}{n}}}{2^{\frac{n-1}{n}}} dx =$$

$$= \lim_{x \rightarrow \infty} \int_3^{\infty} \frac{(2x-1)(x+1)^2}{2} dx = \frac{1}{2} !$$

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