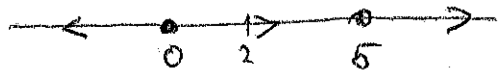
1. (A) a fázisegyenes: 

Egyértelműség $\Longrightarrow$ $0 < x(t) < 5 \quad \forall t \in (\alpha,\beta)$

Folytathatósági tétel $\Longrightarrow$ $(\alpha,\beta) = (-\infty,\infty)$.

(B) $x \equiv 0$ megoldás + egyértelműség van $\Longrightarrow$ $(-\infty,\infty)$ a max. létezési intervallum.

(C) $\int_0^t \left(-\frac{x'(s)}{x^2(s)}\right) ds = \int_0^t 1\, ds \Longrightarrow \frac{1}{x(t)} - \frac{1}{2} = t \Longrightarrow x(t) = \frac{1}{t+\frac{1}{2}}$

$\left(-\frac{1}{2},\infty\right)$ a max. lét. int.

2. (a) homogén $\Longrightarrow$ $u = \frac{x}{t}$: $\quad x' = u'\cdot t + u = \frac{1}{u} + u \Longrightarrow$

$u'\cdot t = \frac{1}{u} \Longrightarrow u'u = \frac{1}{t} \Longrightarrow \frac{1}{2}(u^2)' = \frac{1}{t}$

$\Longrightarrow \frac{1}{2}u^2(t) = \log|t| + k \Longrightarrow u(t) = \pm\sqrt{2\log|t| + c}$

$\Longrightarrow x(t) = \pm\, t\sqrt{2\log|t| + c}$

(b) Bernoulli $\Longrightarrow$ $u = \frac{1}{x^2}$, $u' = -\frac{2x'}{x^3} \Longrightarrow u' + \frac{2}{t}u = \frac{2}{t^3}$

$\Longrightarrow (u\cdot t^2)' = \frac{2}{t} \Longrightarrow u(t)\cdot t^2 = 2\log|t| + c$

$\Longrightarrow x(t) = \pm\frac{t}{\sqrt{2\log|t| + c}}$

3. $x(t)$: a leves hőmérséklete t-ben

$x'(t) = -k\,[x(t) - 20]$

$x(0) = 100$, $x(10) = 60$, $x(T) = 25$, $T = ?$

$x(t) = c\,e^{-kt} + 20$

$t=0$: $\quad 100 = c + 20 \Longrightarrow c = 80$

$t=10$: $\quad 60 = 80\cdot e^{-10k} + 20 \Longrightarrow e^{10k} = 2$

$t=T$: $\quad 25 = 80\cdot e^{-T\cdot k} + 20 \Longrightarrow e^{Tk} = 16 = 2^4$ $\Big\} \Longrightarrow$

$\Longrightarrow e^{Tk} = (e^{10k})^4 = e^{40k} \Longrightarrow \underline{T = 40}$

4. $x' = (1+x^2)e^t$, $x(0) = 0$ $\qquad \frac{x'(t)}{1+x^2(t)} = e^t$

$\int_0^{x(t)} \frac{du}{1+u^2} = \int_0^t e^s ds = e^t - 1 \qquad \int \frac{du}{1+u^2} = \operatorname{arctg} u + c$

$\operatorname{arctg} x(t) - 0 = e^t - 1 \Longrightarrow x(t) = \operatorname{tg}(e^t - 1)$

$(\alpha,\beta) = ?$, $0 \in (\alpha,\beta) \Longrightarrow e^t - 1 \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \Longleftrightarrow$

$\underline{t \in \left(-\infty, \log\left(\frac{\pi}{2} + 1\right)\right) = (\alpha,\beta)}$

5. x(t): sómennyiség kg-ban

$x(0)=0$

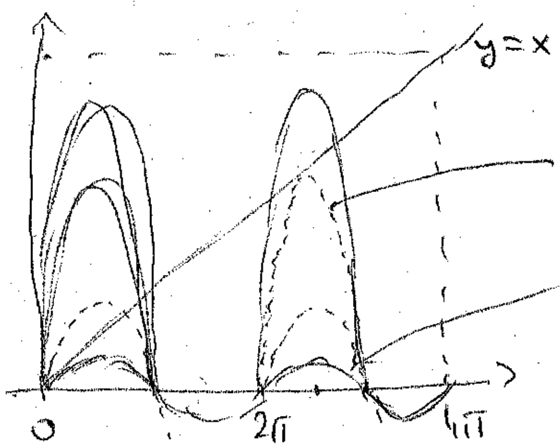
$x'(t) = 2\cdot 0{,}3 - \frac{2}{10}x(t) = 0{,}2\,(3-x(t))$

$x(t) = c\,e^{-0{,}2t} + 3$

$t=0:\quad 0 = c+3 \Rightarrow c=-3 \Rightarrow x(t) = 3\left(1-e^{-0{,}2t}\right)$

$x(5) = \underline{3\left(1-e^{-1}\right)}$ kg

6. $x' = -x + a\sin x \qquad a\in[0,4\pi]$



$0\le a\le 1$:

$1<a<a^*$:

$a=a^*$

$a^*<a\le 4\pi$