Feladatok

Oldjuk meg az alábbi feladatokat!

(1) $$\frac{dx}{dt} + \frac{1}{t}x = t^2, \quad x(1) = 0.$$

(2) $$\frac{dx}{dt} + t^2 x = t^2, \quad x(0) = 1.$$

(3) $$\frac{dx}{dt} + x = \cos t, \quad x(0) = 0.$$

(4) $$\frac{dx}{dt} + tx = t^3, \quad x(0) = 1.$$

(5) $$\frac{dx}{dt} + tx = x^3, \quad x(0) = 1.$$

(6) $$\frac{dx}{dt} + (\tan t)x = \cos t, \quad x(0) = 1.$$

(7) $$\frac{dx}{dt} + (\sec t)x = \cos t, \quad x(0) = 1.$$

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(1) $$\frac{dx}{dt} = \frac{x^2+1}{t^2+1}, \quad x(0) = 0.$$

(2) $$\frac{dx}{dt} = (x^2-1)e^t, \quad x(0) = 1.$$

(3) $$\frac{dx}{dt} = e^{x-t}, \quad x(0) = 0.$$

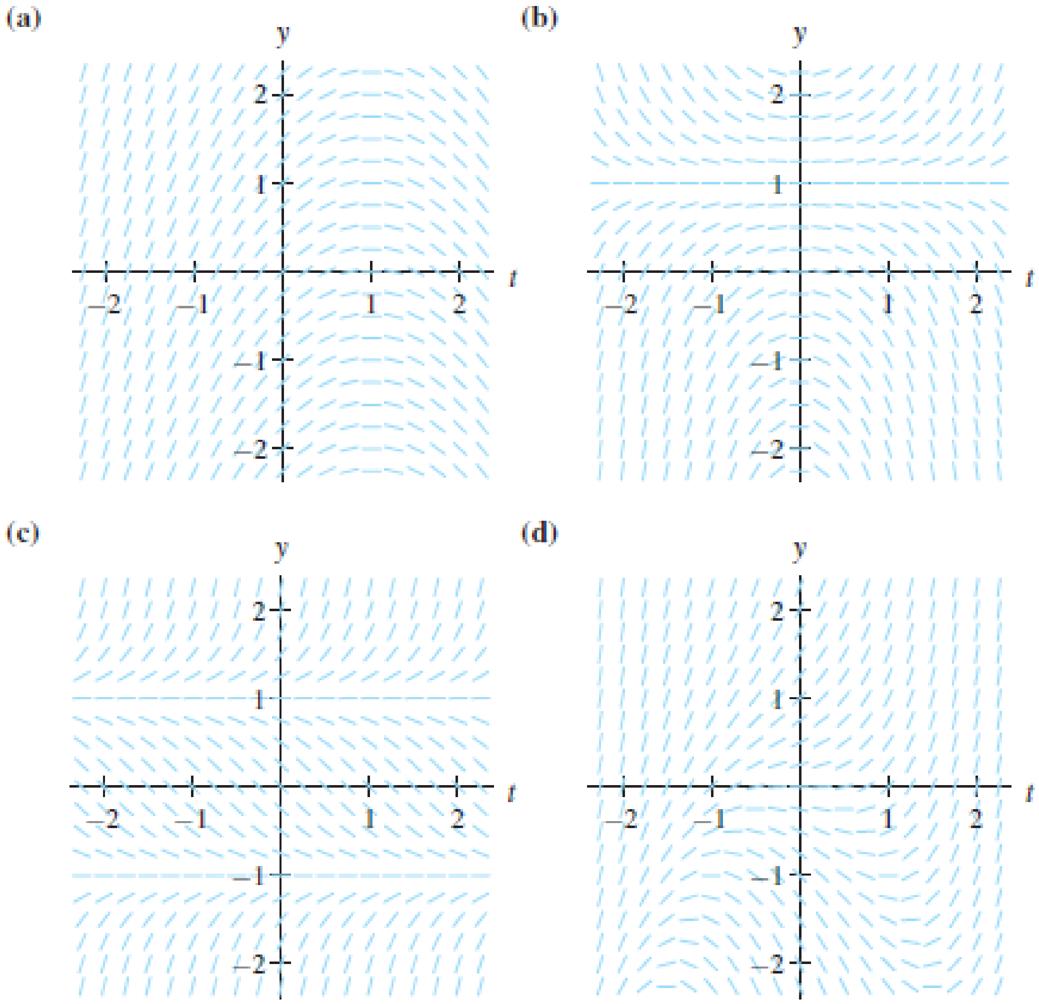
(4) $$\frac{dx}{dt} = \sqrt{x^2+1}, \quad x(0) = 0.$$

(5) $$\frac{dx}{dt} = \frac{xt}{x^2+t^2}, \quad x(1) = 1.$$

Adott 8 egyenlet és 4 iránymező. Határozzuk meg, hogy melyik iránymezőhöz melyik egyenlet tartozik! Miért? (Itt nem kell számolni.)

(i) $\dfrac{dy}{dt} = t - 1$ (ii) $\dfrac{dy}{dt} = 1 - y^2$ (iii) $\dfrac{dy}{dt} = y - t^2$ (iv) $\dfrac{dy}{dt} = 1 - t$

(v) $\dfrac{dy}{dt} = 1 - y$ (vi) $\dfrac{dy}{dt} = y + t^2$ (vii) $\dfrac{dy}{dt} = ty - t$ (viii) $\dfrac{dy}{dt} = y^2 - 1$



További feladatok:

1.2.1.- Find the solution y to the IVP
$$y' = -y + e^{-2t}, \quad y(0) = 3.$$

1.2.2.- Find the solution y to the IVP
$$y' = y + 2te^{2t}, \quad y(0) = 0.$$

1.2.3.- Find the solution y to the IVP
$$t\,y' + 2\,y = \frac{\sin(t)}{t}, \quad y\Big(\frac{\pi}{2}\Big) = \frac{2}{\pi},$$
for $t > 0$.

1.2.4.- Find all solutions y to the ODE
$$\frac{y'}{(t^2+1)y} = 4t.$$

1.2.5.- Find all solutions y to the ODE
$$ty' + n\,y = t^2,$$
with n a positive integer.

1.2.6.- Find all solutions to the ODE
$$2ty - y' = 0.$$
Show that given two solutions y_1 and y_2 of the equation above, the addition $y_1 + y_2$ is also a solution.

1.2.7.- Find every solution of the equation
$$y' + t\,y = t\,y^2.$$

1.2.8.- Find every solution of the equation
$$y' = -x\,y = 6x\,\sqrt{y}.$$

1.3.1.- Find all solutions y to the ODE
$$y' = \frac{t^2}{y}.$$
Express the solutions in explicit form.

1.3.2.- Find every solution y of the ODE
$$3t^2 + 4y^3y' - 1 + y' = 0.$$
Leave the solution in implicit form.

1.3.3.- Find the solution y to the IVP
$$y' = t^2y^2, \quad y(0) = 1.$$

1.3.4.- Find every solution y of the ODE
$$ty + \sqrt{1+t^2}\,y' = 0.$$

1.3.5.- Find every solution y of the Euler homogeneous equation
$$y' = \frac{y+t}{t}.$$

1.3.6.- Find all solutions y to the ODE
$$y' = \frac{t^2+y^2}{ty}.$$

1.3.7.- Find the explicit solution to the IVP
$$(t^2 + 2ty)\,y' = y^2, \quad y(1) = 1.$$

1.3.8.- Prove that if $y' = f(t, y)$ is an Euler homogeneous equation and $y_1(t)$ is a solution, then $y(t) = (1/k)\,y_1(kt)$ is also a solution for every non-zero $k \in \mathbb{R}$.

1.4.1.- Consider the equation

$$(1+t^2)\,y' = -2t\,y.$$

(a) Determine whether the differential equation is exact.

(b) Find every solution of the equation above.

1.4.2.- Consider the equation

$$t\cos(y)\,y' - 2y\,y' = -t - \sin(y).$$

(a) Determine whether the differential equation is exact.

(b) Find every solution of the equation above.

1.4.3.- Consider the equation

$$y' = \frac{-2 - y\,e^{ty}}{-2y + t\,e^{ty}}.$$

(a) Determine whether the differential equation is exact.

(b) Find every solution of the equation above.

1.5.1.- A radioactive material decays at a rate proportional to the amount present. Initially there are 50 milligrams of the material present and after one hour the material has lost 80% of its original mass.

(a) Find the mass of the material as function of time.

(b) Find the mass of the material after four hours.

(c) Find the half-life of the material.

1.5.2.- A tank initially contains $V_0 = 100$ liters of water with $Q_0 = 25$ grams of salt. The tank is rinsed with fresh water flowing in at a rate of $r_i = 5$ liters per minute and leaving the tank at the same rate. The water in the tank is well-stirred. Find the time such that the amount the salt in the tank is $Q_1 = 5$ grams.

1.5.3.- A tank initially contains $V_0 = 100$ liters of pure water. Water enters the tank at a rate of $r_i = 2$ liters per minute with a salt concentration of $q_1 = 3$ grams per liter. The instantaneously mixed mixture leaves the tank at the same rate it enters the tank. Find the salt concentration in the tank at any time $t \geqslant 0$. Also find the limiting amount of salt in the tank in the limit $t \to \infty$.

1.5.4.- A tank with a capacity of $V_m = 500$ liters originally contains $V_0 = 200$ liters of water with $Q_0 = 100$ grams of salt in solution. Water containing salt with concentration of $q_i = 1$ gram per liter is poured in at a rate of $r_i = 3$ liters per minute. The well-stirred water is allowed to pour out the tank at a rate of $r_o = 2$ liters per minute. Find the salt concentration in the tank at the time when the tank is about to overflow. Compare this concentration with the limiting concentration at infinity time if the tank had infinity capacity.

1.9. Consider the IVP

$$\begin{cases} x_1' = x_1 + \dfrac{1}{t-1}x_2^2, & x_1(t_0) = a_1 \\ x_2' = t - x_1 x_2^{1/3}, & x_2(t_0) = a_2. \end{cases}$$

Based on the existence and uniqueness theorems, what can you say about the local existence and uniqueness of the solutions of the IVP for the following values of t_0, a_1, and a_2? Justify your answer.

(a) $t_0 = 2,\ a_1 = 1,\ a_2 = -1;$
(b) $t_0 = 2,\ a_1 = 1,\ a_2 = 0;$
(c) $t_0 = 1,\ a_1 = 1,\ a_2 = -1.$

1.10. For which values of t_0, a_1, a_2 and a_3, does the IVP

$$y''' = (\sin t)e^y + \frac{(y'')^{5/3}}{\cos t} + (y')^{1/3}, \quad y(t_0) = a_1, y'(t_0) = a_2, y''(t_0) = a_3,$$

have a solution and have a unique solution, respectively? Justify your answers.

1.16. Discuss the existence, uniqueness, and the maximal interval of existence of the solution of the IVP

$$x' = \ln t + \frac{x}{x^2+1}, \quad x(1) = 0.$$

1.17. For each of the following IVPs

(a) $x' = \dfrac{t^2 x}{1+e^x},\ x(t_0) = x_0,$
(b) $x' = \sin(tx)x + t,\ x(t_0) = x_0,$
(c) $x' = e^t \dfrac{x^3}{1+x^2},\ x(t_0) = x_0;$

show that the solution exists on the interval $[t_0, \infty)$ by changing it to an integral equation and using a Gronwall inequality.